Party Formation and Policy Outcomes under Different Electoral Systems *

Massimo Morelli

Ohio State University and
Institute for Advanced Study, Princeton

First draft: August 1998; This draft: December 2001.

Abstract

I introduce a model of representative democracy that allows for strategic parties, strategic candidates, strategic voters, and multiple districts. If the distribution of policy preferences is sufficiently similar across districts and sufficiently close to uniform within districts, then the number of effective parties is larger under Proportional Representation than under Plurality Voting (extending the Duvergerian predictions), and both electoral systems determine the median voter's preferred policy outcome. However, for more asymmetric distributions of preferences the comparative results are very different; the Duvergerian predictions can be reversed; compared with the median voter's preferred policy, the outcome with Proportional Representation can be biased only towards the center, whereas under Plurality Voting the policy outcome can be anywhere. The sincere vs. strategic voting issue is welfare irrelevant, but sincere voting induces more party formation.

Keywords: Electoral Systems, Voting Recommendations, Multiple Districts, Endogenous Candidates, Strategic Parties.

---

*I wish to thank P. Bolton, J. Chan, V.V. Chari, S. Coate, A. Dixit, J. Dow, S. Feldmann, G. Frechette, R. Kranton, F. Maniquet, E. Maskin, R. Myerson, P. Penelle, A. Rubinstein and three anonymous referees for their suggestions. I would also like to thank Deutsche Bank for sponsoring me as a member of the Institute for Advanced Study. Comments by seminar participants at Cornell, Johns Hopkins, NYU, IAS, Chicago and Penn are gratefully acknowledged. The usual disclaimer applies.
E-mail: morelli@ias.edu.

1 Introduction

In political science the most famous predictions about the effects of electoral systems are the so called "Duverger's law" and "Duverger's hypothesis" (Duverger 1954).[1] These are informal observations/predictions, which can be summarized as follows: *Duverger's law* states that under Plurality Voting there are forces leading the number of effective parties to be no greater than two; *Duverger's hypothesis* states that under proportional systems there is a tendency to multipartyism. Hence the *Duvergerian comparative prediction* is that the number of effective parties is larger in an election when a proportional system is used than under majoritarian systems. These informal predictions were about elections in a single or unified district. There have been several formalizations of Duverger's law,[2] showing that it can be derived from the rational choice of strategic voters. Not as much work has been devoted to Duverger's hypothesis, but there are some indications that strategic voting can lead to a reduction in the number of effective parties also under Proportional Representation.[3] In any case, all the existing formal models that relate to the Duvergerian predictions stick to the single-district world; do not distinguish the role of candidates from that of parties, both often taken as given; and let all the action be at the voting stage. One of the contributions of this paper is to provide a framework where the Duvergerian predictions can be studied even when the electorate is divided in multiple districts and candidates and parties are separate entities. The party structure as well as the type composition of the pool of candidates are endogenous and play different roles.

Any serious comparison of electoral systems in representative democracies requires a characterization of the interplay of strategic voters, strategic parties, and strategic candidates, within and across districts. When this is done, the equilibrium outcomes of representative democracy turn out to be much less sensitive to the assumptions on voters' behavior than in the existing literature. Strategic parties and endogenous candidates can *"substitute"* for the coordination of voters' strategies. Endogenous candidacy is necessary (see Dutta, Jackson,

[1]The term Duverger's law was actually introduced by Riker (1982). A lucid discussion and empirical evidence on these two predictions can be found in Cox (1997).

[2]See for example Fey (1997), Feddersen (1992), Palfrey (1989).

[3]Leys (1959) and Sartori (1967) were the first to point out that Duverger's hypothesis could be falsified in some circumstances because of strategic voting. De Sinopoli and Iannantuoni (2001) formalize this point in a special spatial model.

and Le Breton 2001) to appropriately compare voting procedures, but it is also sufficient, most of the time, to determine rational outcomes even when the voters are not strategic. This is a conceptual point that goes beyond the results of this paper on the comparison of electoral systems: Even though sincere voting *strategies* may not be rational, the equilibrium voting *actions* may well be sincere when candidates are endogenous.

Beside the methodological innovations on how to extend the analysis of Duvergerian predictions to multi-district representative democracies and the conceptual point on the sincere vs. strategic voting issue, this paper also provides some simple welfare analysis of the most used electoral systems. In particular, the policy outcome of representative democracy under Proportional Representation (PR) and Plurality Voting (PV) is compared with the median voter's preferred policy.

The summary description of the model is as follows. For each type of policy preference in the population there is a (homogeneous) party to begin with, with a set of politicians and a party leader. If some party leaders agree on some policy compromise, then they can form a heterogeneous party with that compromise as policy platform; otherwise all parties will simply have homogeneous sets of politicians. After the party structure is determined, the politicians decide whether to run or not (endogenous candidacy). Voting is the third stage of the game. The electoral system determines a mapping from the election results (i.e., distributions of votes) to a distribution of seats in a parliament, which then determines the policy by majority rule.

The primary role of parties (homogeneous or heterogeneous) is that they provide a *co-ordination* device to voters *during* the elections. When sincere voting is not an equilibrium and there are many ways in which voters could vote, the party leaders help their voters to coordinate their strategies.[4] In addition, a heterogeneous party can provide a *commitment* device to its politicians *before* elections. In contrast with the standard electoral competition models where candidates only care about winning office and hence can credibly commit to any policy platform,[5] voters here know the policy preferences of all politicians, and they can believe that a politician is going to pursue a policy platform different from her own preferred

[4] Simply think of party leaders going on TV to explicitly say what they want their voters to do.

[5] See Lizzeri and Persico (2001) for a specific electoral competition model of this type with interesting implications for a comparison between proportional and pluralitarian systems. See Wittman (1977) and Alesina (1988) for a discussion of credibility problems when candidates have policy preferences.

one only if the announced policy platform corresponds to a policy compromise that had been agreed upon within her party.[6] The analysis will emphasize that the role of parties as a coordination device is often crucial, whereas the commitment-device role is rarely important in the presence of endogenous candidacy. Heterogeneous parties form only when voters are expected to be sincere and endogenous candidacy is either missing or ineffective.

In order to obtain comparative results I will characterize the equilibria for every distribution of policy preferences. Under PR there are multiple candidates in each district, whereas under PV every equilibrium will display a unique running candidate in each district. If the distribution of policy preferences is sufficiently similar across districts and sufficiently close to uniform within districts, then the Duvergerian comparative prediction turns out to extend to the multi-district world studied here: the number of active and effective parties is higher under PR than under PV. On the other hand, if the distribution of policy preferences is somewhat polarized or skewed in some districts and sufficiently dissimilar across districts, then the Duvergerian comparative prediction can be *reversed*, i.e., a larger number of active and effective parties can be expected under PV than under PR. The multiplicity of parties in India in spite of its PV system is not an anomaly, it simply follows from the extreme differences among Indian states in terms of political preferences (including of course religious cleavages).[7] On the PR side, the exceptions to multipartyism (like Austria, Australia, Ireland and Germany) can also be explained in this framework.

Politicians (potential candidates) care both about the private benefits of being elected (e.g., "ego rents") and about the policy outcome. Both dimensions are important for the determination of the incentives to run. The paper will show, however, that the balance between private benefits from election and policy preferences may matter for the equilibrium party structure *only* under sincere voting and PR: In that case, if an extreme type has the

[6] Other papers that studied different issues related to the role of parties in rational models of representative democracy are Baron (1993), Jackson and Moselle (2001), Caillaud and Tirole (1999), and Riviere (1998). Baron (1993) views parties as coalitions of voters, each voting for one of three exogenously given candidates. Jackson and Moselle (2001) study party-like behavior in the legislature, with no explicit party formation stage. Caillaud and Tirole interpret parties as information intermediaries that select high quality candidates. Riviere (1998) views parties as a way to help candidates to share the candidacy costs. Alesina and Spear (1988) and Harrington (1992) pointed out the role of parties as long run players that try to discipline their candidates, who have a much shorter horizon.

[7] For a comparison of PV systems like India and the United States, see Chhibber and Kollman (1998).

relative majority of preferences in the country (in a way that it could obtain the majority of seats in the parliament under sincere voting), then the only way for the other types to avoid such an extreme outcome is to have only a subset of them run, so that sincere voters, who choose the closest candidates to their type, would be induced to coordinate. If the "ego rents" are small compared to the policy gains obtainable this way, some types of candidates will decide not to run in order to allow that coordination. On the other hand, if the private benefits from being elected outweigh the policy considerations, then the only way to achieve that coordination is to form a heterogeneous party at the beginning. Under strategic voting, instead, these considerations are irrelevant, because candidates will anticipate that coordination will occur anyway at the voting stage.

As far as policies are concerned, for distributions of preferences close "enough" to uniform in every district, the median voter's preferred outcome is the unique policy outcome under both PR and PV. Under PR there are distributions of preferences in the whole country such that the policy outcome can be more "centrist" than what the median voter wants, but there is no distribution of policy preferences that would make the policy outcome diverge from the median voter's preferences in the opposite direction. Under PV, on the other hand, the policy outcome can be more centrist but also more extreme. Hence with non-linear utility functions welfare would turn out to be always higher under PR, and in any case a higher variance in policy outcomes can be expected over time in countries using a PV system.

The paper is organized as follows: Section 2 describes the model; Section 3 contains the complete characterization of equilibrium, and Section 4 draws the main lessons from the characterization results. Section 5 highlights some robustness issues and generalizations. Section 6 concludes and emphasizes the contribution to the literature.

2 The Model

Consider a representative democracy divided in three districts, indexed by $l = 1, 2, 3$. There are three types of citizens, identified by their position $t_i,\ i = L, M, H$, on a unidimensional policy space (single-peaked policy preferences). To use the simplest normalization of such a policy space, let $t_L = 0,\ t_H = 1$, and $0 < t_M \leq \frac{1}{2}$.[8] In each district l there is a continuum

[8]The choice to make type M be closer to type L than to type H is obviously without loss of generality. It simplifies notation though: when I will have to distinguish between the closer of the two extreme types and

of voters of each type.[9] To avoid studying multiple cases, let's assume that all districts have the same measure of voters, normalized to $\frac{1}{3}$ per district. The set P of politicians (potential candidates) is exogenous.[10] However, the set of actual candidates will be endogenous.

The policy outcome t^* is decided via majority rule by the elected parliament, composed of three elected members. The utility function of a citizen of type i is simply $-|t_i - t^*|$. Any politician has the same utility function as any other citizen of the same type, but an additional motivation to run (beside the possibility to affect the policy outcome) derives from a non-transferable private benefit from being elected, π. Some interesting results will come from studying the effect of changing the relative importance of private benefits from election and policy preferences.

Having introduced all the ingredients, let's now turn to describe the representative democracy game.

2.1 Stage 1: Party Formation

Before the game starts, the citizens of each type i are represented by a party A_i, which has an exogenous set of politicians P_i taken from the set of citizens of type i. Only politicians can be candidates. The exogenous set of politicians, $P = P_L \cup P_M \cup P_H$, contains for simplicity only nine members, one of each type in each district ($\#P_i = 3\ \forall i$). Each homogeneous party A_i has a *leader*. For simplicity let's assume that the party leaders are not in P, i.e., they are not potential candidates themselves. This way the party leader's objective at the party formation stage is the same as that of any other private citizen of her type.[11] Denote by λ_i the party leader of party A_i. The three leaders play a party formation game as follows.

1. First λ_L and λ_H simultaneously make offers to λ_M, and each of the two offers is con-

the one further away I will be able to call them just L and H respectively.

[9] Nothing changes if one wants to use a finite number of citizens.

[10] In an earlier version of the paper (available upon request) the number of districts, the number of types, and the relative size of districts and types were left unspecified and general, and the set of politicians was endogenous. However, since the results for three districts, three types, and exogenous politicians are qualitatively similar and clearer, the lack of generality is not important.

[11] Think of the party leader as a principal representing the voters of the same type, and think of politicians as agents. The potential conflict of interest between the party leadership and the candidates is well documented. See, among others, Caillaud and Tirole (1999).

stituted by a policy proposal $\tau_i \in [0, 1]$.

2. Then λ_M chooses a response $r \in \{0, L, H\}$, where 0 means that no offer is accepted and $r = i$ means that the offer by λ_i is accepted $(i = L, H)$. In case of indifference, $r = 0$.[12]

Each profile (τ_L, τ_H, r) determines a party structure in the following simple way:

1. If $r = 0$, then the party structure remains $\sigma_0 \equiv \{A_L, A_M, A_H\}$;

2. If $r = L$, then $\sigma_L \equiv \{A_L \cup A_M, A_H\}$;

3. If $r = H$, then $\sigma_H \equiv \{A_L, A_M \cup A_H\}$.

Thus, the endogenous number of parties is $n(r) = 2$ if $r = i \in \{L, H\}$ and $n(r) = 3$ if $r = 0$. I will use the index j when referring to a generic party of a party structure σ.

The corresponding vector of party positions varies as follows:

1. $\tau(r = 0) = (t_L, t_M, t_H)$;

2. $\tau(r = L) = (\tau_L, t_H)$;

3. $\tau(r = H) = (t_L, \tau_H)$.

Every politician k in party $j \in \sigma$ will be evaluated by voters on the basis of the endogenous τ_j (which, as just described, equals t_k if j is a homogeneous party). In other words, politicians cannot be associated to a policy platform different from their ideological one unless they belong to a heterogeneous party $A_i \cup A_M$ $(i = L, H)$ that agreed on a policy compromise τ_i, in which case all the politicians of $A_i \cup A_M$ are all identified as having the same policy platform τ_i.[13]

[12] This tie-breaking rule is consistent with any assumption that one could make about positive costs of forming heterogeneous parties.

[13] The assumption that a policy compromise in a heterogeneous party $A_i \cup A_M$ $(i = L, H)$ is perfectly credible to voters is made for simplicity, but credibility could be easily endogenized (see Section 5). I will also argue that the results would certainly extend to more symmetric party formation game forms.

2.2 Stage 2: Candidacy

For every outcome of stage 1, i.e., for every $\sigma \in \{\sigma_0, \sigma_L, \sigma_H\}$ and for every τ, the nine politicians have to decide whether to run or not. For simplicity, I assume that they move sequentially, and that the politicians of type M are the first three to move; then those of type L and those of type H.[14] The decision of politician k is denoted by $I_k \in \{0, 1\}$, where 1(0) indicates the decision to run(not to run). The endogenous number of candidates is then $y = \sum_{k=1}^{9} I_k$. The set of endogenous candidates will be denoted by Y, and the set of endogenous candidates in district l will be denoted by Y^l. When all the three politicians of the three types in district l decide to run I will use the simple notation $Y^l = P^l$.

Candidacy involves a small cost c, with $0 < c < \pi/3$. A heterogeneous party could have more than one candidate per district, but a strictly positive candidacy cost implies that this never happens in equilibrium. Note that $\pi > c$ would be sufficient to guarantee that $y^l \geq 1 \ \ \forall l$, but I require $\pi > 3c$ for reasons that will become clear later.

2.3 Stage 3: Voting

Each voter of each district l has to choose among the candidates in Y^l. Voters have single-peaked preferences on the policy space $[0, 1]$ (with peaks 0, t_M, and 1). The set of distributions of preferences is $D = \left\{ \{\mu_i^l\}_{i=L,M,H;l=1,2,3} \ : \ \sum_i \mu_i^l = 1 \ \forall l \right\}$, where μ_i^l is the (strictly positive) fraction of voters of type i in district l.[15] I will also use the notation $\mu_i = \sum_l \mu_i^l / 3$ to denote the fraction of the country's population who have t_i as most preferred policy. A specific distribution will be denoted by $d \in D$.

I will consider two different scenarios: sincere voting and a simple form of strategic voting. With a continuum of voters sincere voting is actually an undominated Nash equilibrium, as any other voting profile, because no voter can ever be pivotal. However, in the presence of parties it is realistic that voters can coordinate (or be coordinated), and hence they can behave as a finite number of players, in which case sincere voting is not necessarily a Nash

[14] The sequential choice can be replaced by a simultaneous move game, but one would then have to add some refinements to avoid multiplicity problems. The choice of having the median type players move first is motivated by the desire to avoid multiple tedious cases in the main text. However, as shown in Section 5, the results do not depend on the order of play.

[15] Since μ_i^l is a fraction, the sum of the three fractions must be one, even though each district has measure $\frac{1}{3}$ of voters.

equilibrium behavior. In order to simplify the description of the two voting scenarios, I will ignore the case in which $Y^l = \emptyset$ for some l, since this is ruled out as an equilibrium by having $\pi > c$.

Sincere voting implies that each citizen votes for the candidate of the party with the closest position among those in her district. In other words, each voter v in district l casts her vote for a candidate k such that

$$k \in \arg\min_{k \in Y^l} |t_v - \tau_k|.^{16}$$

For any set of candidates Y, any distribution of preferences d, and any set of policy positions $\{\tau_k\}_{k\in Y}$, $z_s(Y, d, \{\tau_k\}_{k\in Y})$ will denote the corresponding sincere voting profile. The specification of $\{\tau_k\}_{k\in Y}$ can be dropped when $\sigma = \sigma_0$, since in that case we know that the positions of the three parties are the original ones.

As a simple strategic voting scenario, I consider a *perfect coordination* environment. Think of the $n(r)$ parties as making *"voting recommendations"*. The voters of type i follow the voting recommendation of their party leader if and only if such a recommendation constitutes a best response to the recommendations made by the other party leaders.[17] In other words, the voting recommendations of the $n(r)$ parties have to constitute a Nash equilibrium. When $n = 2$ sincere voting recommendations, i.e., where each party suggests its own candidates, are obviously Nash.[18] The only case that needs formalization is therefore when $n = 3$. In this case each party A_i (or its party leader) chooses a recommendation triplet, $z_i = \{z_i^l\}_{l=1,2,3}$, where each component is a recommendation to a district's voters of type i. Formally, each component of i's recommendation strategy is a function $z_i^l : 2^P \times D \rightarrow Y^l$ that associates to any pair (Y, d) a recommendation $k \in Y^l$. Denoting by $t(z)$ the continuation policy outcome determined by the profile of voting recommendation strategies z, a strategy profile z^* is a

[16]If there is more than one candidate with the same τ, the vote is given to anyone of them with equal probability.

[17]On an off-equilibrium path where a party leader is not making a recommendation that is best response to the other recommendations, the voters of that type are left without guidance or coordination, so in that case we might as well assume for completeness that they vote sincerely, but any other assumption would do.

[18]For example, if the two parties are $A_L \cup A_M$ and A_H, all the citizens of type L and M prefer (at least weakly) any $\tau_L \in [0, t_M]$ to $t_H = 1$. If the two parties are A_L and $A_M \cup A_H$, then there could be values of $\tau_H \in [t_M, 1]$ such that the voters of type M would prefer to vote for L candidates even if sincere voting recommendations remain Nash. But those values of τ_H could never be chosen in equilibrium.

voting equilibrium given a distribution of preferences d if and only if

$$|t(z^*(Y;d)) - t_i| \leq |t(z_i, z^*_{-i}(Y;d)) - t_i| \quad \forall i, \forall z_i$$

for every Y with $y^l \geq 1$. $Z^*(d)$ will denote the set of such equilibrium strategies.[19] The set of voting equilibria for a specific set of candidates will be denoted by $Z^*(Y;d)$.

An equilibrium voting profile z^{**} in Z^* is *Strong* iff there is no coalition of parties $C \in 2^\sigma$ such that

$$|t(z^{**}(Y;d)) - t_i| > |t(z_C, z^{**}_{-C}(Y;d)) - t_i| \quad \forall i \in C, \text{ for some } z_C.$$

$Z^{**}(d)$ will denote the set of strong voting equilibrium strategy profiles, and $Z^{**}(Y;d)$ will be the set of strong voting equilibria for a specific set of candidates.

Definition 1 Sincere voting is *rational* given a specific distribution of preferences d iff $z_s(Y,d) \in Z^*(Y;d) \quad \forall Y : Y^l \neq \emptyset$. It is *strongly rational* iff $z_s(Y,d) \in Z^{**}(Y;d) \quad \forall Y : Y^l \neq \emptyset$.

The distinction between strategies and actions will be important for the evaluation of sincere vs. strategic voting: even in cases in which sincere voting is not rational in the sense of Definition 1 (i.e., as a strategy), the equilibrium actions may well be sincere, since some subgames (characterized by sets of candidates such that sincere voting would not be a continuation equilibrium) are not reached by any equilibrium path. I will discuss this issue in detail when showing the characterization results and in Section 4.3.

2.4 Electoral Systems

An electoral system determines a distribution of seats for every voting outcome: Denoting by v^l_k the number (measure) of votes obtained by candidate k in district l (with $\sum_{k \in Y^l} v^l_k = \frac{1}{3}$), the general form for the mapping into seats for party j can be represented by the function $F_j : [0, 1/3]^y \to \{0, 1, 2, 3\}$ that associates to any voting outcome $\{v^l_k\}_{k \in Y, l=1,2,3}$ a number $F_j \in \{0, 1, 2, 3\}$ (where $\sum_{j \in \sigma} F_j = 3$). The distribution of votes to the candidates in Y, $\{v^l_k\}_{k \in Y, l=1,2,3}$, depends on the voting profile; hence $F^E_j(z(Y,d))$ denotes the number of seats going to party j if the voting profile is $z(Y,d)$ and the electoral system is $E \in \{PR, PV\}$.

Among the various rules used in PR systems to transform votes into seats, a commonly used one is the Hare quota. Recalling that each district l has a measure $\frac{1}{3}$ of voters, the total

[19] Note that abstention is not allowed.

number of votes going to party j is $V_j = \sum_{k \in j, l=1,2,3} v_k^l$. The Hare quota rule assigns the first seat to a party j such that $V_j \geq V_{j'}$ for every other party j'; then, the second seat goes to the party with the largest remainder, where the remainder for j is $V_j - \frac{1}{3}$ and the remainder for any other party j' is $V_{j'}$; the third seat, once again, goes to the party with the largest remainder after subtracting $\frac{1}{3}$ from the total number of votes obtained by the party that got the second seat. Formally, if $\sigma = \sigma_0$, $F_j^{PR} = 3$ iff $V_j - \frac{2}{3} > V_{j'}$ $\forall j' \neq j$; $F_j^{PR} = 2$ if $V_j - \frac{1}{3} > V_{j'}$ for some $j' \neq j$ but $V_j - \frac{2}{3} < V_{j'}$ for some $j' \neq j$; $F_j^{PR} = 1$ $\forall j \in \sigma$ iff $\max_j V_j - \frac{1}{3} < V_{j'}$ $\forall j'$. To determine who obtains a seat if a party obtains less seats than its number of candidates, I assume for simplicity that this assignment is done randomly.[20] With $\pi/3 > c$ the expectation that party j will obtain at least one seat is enough to make a politician of that party run in every district.

The PV system is characterized by the following function:

$$F_j^{PV} = \sum_l \sum_{k \in j} g_k^l$$

where $g_k^l = 1$ if $v_k^l = \max_{k' \in Y^l} v_{k'}^l$ and 0 otherwise. The number of seats going to each party depends on how many districts it wins. There is no need to specify any party assignment rule, because each seat is assigned directly to the candidate with the most votes in the district.

Given an electoral system $E \in \{PR, PV\}$, I will denote by Γ_s^E the representative democracy game under sincere voting and by Γ_z^E the one under the strategic voting scenario.

2.5 Policy, Payoffs, and Effective Parties

For every voting profile $z(Y, d)$ there is a distribution of votes, and for every distribution of votes an electoral system determines a specific distribution of seats. Now it is easy to see that for every distribution of seats there is a unique policy outcome, so that the outcome function $t(z(Y, d))$ used in the definition of an equilibrium of the voting recommendation game is well defined. If the majority of seats is held by politicians with the same policy platform – say τ_j – then the policy outcome is obviously $t^* = \tau_j$. If instead the three seats are held by

[20] If one replaces this assumption with the one that the seats obtained by a party are assigned to the candidates who received the largest number of votes, there may be less candidates in equilibrium, but the substantive results in terms of number of active parties, policy outcome and role of strategic voting are unchanged. Hence I prefer the simpler assumption.

politicians of three different parties with different policy platforms, then pure majority rule applied to this one-dimensional bargaining space guarantees that the outcome is the policy preferred by the median of the three representatives, $t^* = t_M$.

The payoff for voter v is $-|t_v - t^*|$, and the final payoff for politician k that joined party j is $U_k = -I_k c - |t_k - t^*|$ if not elected and $\pi + U_k$ if elected.

Beside the policy and the corresponding citizens' payoffs, an important political outcome of the representative democracy game is the number of parties that are effective or at least active.

Definition 2 A party j is *active* if and only if there is at least one running candidate of that party in the whole country, i.e., iff $\sum_{k \in j} I_k \geq 1$.

Definition 3 A party j is *effective* if and only if there is at least an elected candidate of that party in the whole country, i.e., iff $F_j \geq 1$.

The Duvergerian predictions have to be interpreted not in terms of the actual number n of parties, but in terms of "how many parties have a significant chance of winning at least some seats". This is what Duverger would like to measure under the two electoral systems. However, since this model has complete information, the term "significant chance" does not have any meaning (each party either wins some seats or it does not). The measurement that Duverger would like to see is some intermediate one between the number of parties that obtain seats (effective parties) and the number of parties that have running candidates (active parties). Even though the idea of "significant chance" cannot be captured in this model, it is clear that a party needs at least to be active in order to be counted in any Duvergerian count, hence being active is the minimum requirement; On the other hand, effectiveness as defined here implies effectiveness in the Duvergerian sense, hence Definition 3 is the maximum requirement. I will show that the characterization results in terms of active and effective parties coincide, so that effectiveness in any intermediate sense between the maximum and the minimum requirement must also be implicitly included in the results.

The Reform party in the US has been active but not effective, so it probably should not count in any intermediate Duvergerian count either. The Liberal party in the UK, on the other hand, satisfies the condition for effectiveness, and so do many parties in the Plurality system in India, and hence such parties should be counted.[21]

2.6 Equilibrium

The equilibrium concept for the whole game is basically subgame perfection (SPE), but I will sometimes focus on the subset of SPE that are also strong (i.e., robust to coalitional deviations at the voting recommendation stage), for comparative purposes. Under the sincere voting scenario the voting stage is mechanical, so the strategy profiles just have to include a specification of party formation and candidacy strategies.

Formally, a strategy profile for Γ_s^E is a tuple $(\tau_L, \tau_H; r(\tau_L, \tau_H); \{I_k(\sigma, \tau)\}_{k=1,\ldots,9})$. Existence of SPE for Γ_s^E is not a problem: After the first simultaneous proposal by the two extreme party leaders, the rest of the game is sequential with discrete choices. Thus, to see that existence is guaranteed it is enough to see that at the initial simultaneous proposal stage there are only two possibilities: (1) λ_M is expected to accept an offer if it is close enough to t_M; (2) λ_M is expected to reject all offers. In case 1 the only equilibrium (if at least one of the extreme parties has incentive to make an offer) is with both extreme party leaders offering $\tau = t_M$; in case 2 all offers are equivalent and equally irrelevant.

The formal representation of a strategy profile for Γ_z^E has to include voting strategies: $(\tau_L, \tau_H; r(\tau_L, \tau_H); \{I_k(\sigma, \tau)\}_{k=1,\ldots,9}; \{z_j\}_{j\in\sigma})$. Existence of SPE for Γ_z^E will be clear from the characterization results.

It is worth noting (and will be clear from the analysis) that even though there is a unique profile of voting recommendation *actions* that constitute a strong equilibrium at the subgames where sincere voting recommendations are not Nash, there could be multiple equilibrium voting recommendation *strategies* that are strong, i.e., $Z^{**}(d)$ may contain more than one element. The easiest way to see this is under PV: there are some subgames in which the type of the elected candidate in district l does not matter for the final policy outcome (perhaps because the other two seats are surely going to two candidates of the same party). Therefore any action profile for district l at those subgames would be Nash. This type of multiplicity of equilibria in terms of strategy profiles is irrelevant for policy outcomes and for the party structure, since it arises precisely when voters are indifferent. Hence, when comparing the

[21] Duverger only considered single-district elections, so this extrapolation of his own definitions should be taken with a grain of salt.

equilibrium party structure σ^* and the policy outcome t^* between the two systems I will be able to make the comparison without further refinements, precisely because of the uniqueness of those outcomes in spite of a potential multiplicity of equilibrium strategy profiles.

Also note that this type of multiplicity of strong voting profiles is *not robust* to very reasonable perturbations of the game: imagine, for example, that under PV the elected candidate in district l has also to provide a local public good;[22] assume then that the citizens of type i in district l value more the kind of public good that would be provided by the candidate of type i if elected than the kind of public good that would be provided if the elected candidate were of another type. In particular, assume that this extra-value from having $k \in P_i$ elected is ϵ when compared with an adjacent type and 2ϵ when compared with a non adjacent type. In this case, for any $\epsilon > 0$, sincere voting recommendations remain strong equilibrium recommendations at all the subgames where they are Nash, but typically all the other strong equilibria at those subgames are not robust to this ϵ perturbation. I study the model with $\epsilon = 0$ for simplicity, but nothing would change in the results for $\epsilon > 0$ small enough. As it will be clear later, at the subgames where instead sincere voting recommendations are not Nash, there is a unique strong continuation equilibrium even with $\epsilon = 0$. Thus, the ϵ perturbation argument allows me to select for Γ_z^E the strong equilibrium profile that takes the form of sincere voting recommendations when they are Nash and the form of the unique strong equilibrium at the subgames where they are not.[23,24] In terms of the results of the paper, Section 4.1 is the only one where using the selection of this robust strong equilibrium makes comparisons easier. In all other sections even the non robust equilibria are fine, for the reasons mentioned above about the uniqueness of equilibrium outcomes. However, for simplicity of language let us keep the convention that whenever I will talk about strong equilibrium I will mean the robust one.

[22]Under PR one assumption that would have the same effect is that the local public good in district l is provided by some agent of party i with probability equal to the vote share obtained by party i in district l.

[23]This equilibrium selection is similar in spirit to that of Alesina and Rosenthal (1996).

[24]The fact itself that districts exist is, at least in part, due to the existence of heterogeneous preferences in the country and to the desire that at least local public goods reflect local preferences. So at least when voters are indifferent as far as national politics is concerned, this ϵ robustness check is very important. To see the relevance of local public goods for the comparison of electoral systems, see for example Persson and Tabellini (1999).

2.7 Some Comments on Strategic Parties and Candidates

Having described the representative democracy game, it is a good time to remark that all the stages are necessary if one wants to analyze the comparative questions raised in this paper. Parties are important for at least two reasons: because they may provide a commitment device when policy compromises are mutually beneficial, and because they always serve as coordination devices for their own supporters at the voting stage. However, the next section is going to show that the potential commitment-device role is rarely played in equilibrium, because of endogenous candidacy. The fact that candidates are endogenous plays a major role: I will show that it *substitutes* (most of the time) both the role of strategic parties and that of strategic voters.

Beside the "substitution" results just mentioned, which will be discussed in detail in the next sections, endogenous candidacy is also *necessary* for a valid comparison of electoral systems. Why is that? Because the two electoral systems considered here, as most other existing systems, do not satisfy "candidate stability", as pointed out by Dutta, Jackson, and Le Breton (2001). Intuitively, this means that there are many sets of candidates where at least one candidate is strictly better off by dropping out of the race.[25] Hence, comparing PR and PV keeping the same fixed number of candidates may make *no sense*. In fact, the incentives to run under the two systems are very different, and while under PV there is almost always a unique candidate in every district (a different one depending on d of course), under PR even $Y = P$ is possible. It is only by endogenizing the set of candidates that the comparison between electoral systems, both in terms of policy implications and in terms of the Duvergerian comparative prediction, can be accurate.

3 Characterization of Equilibria

In this section I am going to provide the full characterization of equilibria. These characterization results should be of independent interest, and they will serve the purpose of preparing the ground for the presentation of the three main messages of the paper, which will be discussed in Section 4.

[25]Formally, there always exist Y such that $p_k(Y;d)\pi - c - |t(z(Y;d)) - t_k| < -|t(z(Y \setminus k;d)) - t_k|$ for some $k \in Y$, where $p_k(Y;d)$ would be the probability with which k expects to win a seat given Y.

3.1 Plurality Voting

The voting subgame requires some analysis only under the strategic voting scenario, and only at the nodes where $\sigma = \sigma_0$, since otherwise sincere voting is strongly rational. For any voting subgame with any candidates set Y, there always exists at least one Nash equilibrium profile of voting recommendations. Sincere voting is an equilibrium in district l given any $Y^l \neq P^l$.[26] If $Y^l = P^l$ but district l is "not pivotal" given what happens in the other districts, i.e., when the policy outcome is not affected by the voting behavior in district l, then all voting recommendations are Nash. If $Y^l = P^l$ but district l is pivotal, then sincere voting can be an equilibrium behavior only if $\mu_i^l > \frac{1}{2}$ for some type i. If on the other hand $\mu_i^l < \frac{1}{2}\ \ \forall i$, then there are only two possible equilibria: one where the median type voters help the type L candidate to win rather than voting sincerely,[27] and one where the type L voters vote for the type M candidate. Formally, the former recommendation equilibrium is characterized by $z_M^l(P^l) = L$, while the latter is characterized by $z_L^l(P^l) = M$. However, it is easy to see that only the latter is strong. All these observations about the voting subgames will be useful to prove the characterization results.

Let $D_a \equiv \{d : \mu_i^l > \frac{1}{2}, \mu_i^{l'} > \frac{1}{2} \text{ for some } i, l, l'\}$ denote the set of distributions of preferences such that a type i has the absolute majority of preferences in at least two districts. If $d \in D_a$ then no analysis is needed. Both sincere and strategic voting yield the same policy outcome $t^* = t_i$ without formation of heterogeneous parties, in every equilibrium. This implies that in such situations at least one party will never have a chance to get a seat.

Remark 1 *For every $d \in D_a$, under Plurality Voting there are at most two effective parties and, generically, at most two active parties.*[28]

[26] If $y^l = 2$ then consider first the cases where the politician of type M is in Y^l. In these cases it is clear that the voters whose type is not represented by any candidate can never lose by voting for the candidate of type M (i.e., sincere voting). If Y^l does not contain a candidate of type M, then it is equally clear that the voters of the median type can never lose by voting for the candidate of type L, which constitutes sincere voting since the median position is closer to t_L than to t_H.

[27] For example, suppose that in the other two districts the seats go to party M and party H. Then if $\mu_H^l = \max_j \mu_j$ there is no profitable deviation for the median type voters from the proposed voting profile where they support L.

[28] Under sincere voting there is a non generic case where all the three parties are active, namely when some type i has the absolute majority in two districts and the other two types tie in the third district. Under

I will now demonstrate that the sincere vs. strategic voting issues are *irrelevant* also with any other distribution of preferences.

Proposition 1 *For any distribution of preferences in $D \setminus D_a$, Γ_s^{PV} and Γ_z^{PV} have the* same *equilibrium outcomes:*

(I) The equilibrium policy outcome is always t_M and no heterogeneous party ever forms;

(II) There is always only one running candidate per district;

(III) The running candidate of district l is of type $i = L, H$ if and only if $\mu_i^l > \frac{1}{2}$.

Proof.

- *Sincere voting.* If $\mu_i^l > \frac{1}{2}$ for some i, then of course only a candidate of type i has incentive to run (sufficiency in (III)). So (III) can be proved by showing that, whenever $d \in D \setminus D_a$ is such that $\mu_i^l < \frac{1}{2}$ for both $i = L$ and $i = H$, the unique candidate in district l is of type M. To see this, note that if $\mu_i^l < \frac{1}{2}$ for both $i = H$ and $i = L$, then the politician of type M runs and is sure to win unless both the other two politicians run. But the one of them with less preferences has no incentive to run since she would lose anyway. Given this, even the extreme type with the relative majority of preferences decides not to run, anticipating that, if she did so, all the other votes (absolute majority, composed of the votes of the median type voters plus those of the other extreme type voters) would go to the median candidate. Hence the median candidate runs uncontested even if there are very few people with the median preference. (II) follows immediately. Moreover, in $D \setminus D_a$ the median party always obtains at least a seat if $\sigma = \sigma_0$, and no other party can obtain more than two seats by construction, hence majority rule implies $t^* = t_M$, which implies $r = 0$ for every pair of offers (τ_L, τ_H) (recall that $r = 0$ is chosen in case of indifference).

- *Strategic voting.* Even though it is still obvious that $\mu_i^l > \frac{1}{2}$ is sufficient to have i as a unique running type in district l, with strategic voting one wonders whether this should remain necessary. To see that it is, simply note that if $\mu_i^l < \frac{1}{2}$ $\forall i$, then the politician of type M always runs, anticipating that the type L politician will then decide to stay out

strategic voting there is not even this non generic case.

because if she entered the type H would then optimally stay out and make the median type win anyway. (I) and (II) follow as well. **QED.**

Proposition 1 shows that if $d \in D \setminus D_a$ then in any equilibrium: the policy outcome is the one preferred by the median type; there are only three running candidates; and in each district the equilibrium running candidate is of some extreme type if and only if it has the absolute majority of preferences in that district. The intuition for the irrelevance of the voting scenario is that $y^l = 3$ would be the only subgame where sincere voting could fail to be Nash, but that subgame is never reached by any equilibrium path.

Note that in each district l the unique running candidate is always of the same type as the median voter of that district. In fact, if $\mu_i^l > \frac{1}{2}$ for $i = H$ or $i = L$, then the median voter of district l is of type i; Otherwise the median voter of district l is of type M. Hence, Proposition 1(III) guarantees that the running candidate of district l is always of the district l median voter's preferred type. Therefore the policy outcome for the whole country is the median of the median voters' positions of the three districts.

Corollary 1 *Under Plurality Voting the policy outcome is always the median of the median voters' positions of the three districts.*

3.2 Proportional Representation

With Proportional Representation it is convenient to study sincere voting and strategic voting separately. Recall that $\mu_i = \sum_l \mu_i^l / 3$.

Proposition 2 *In the game Γ_s^{PR}, the equilibrium party structure can be characterized as follows:*

(I) If $\pi - c < t_M$, then $\sigma^* = \sigma_0$ in every equilibrium.

(II) If $\pi - c \geq t_M$, then:

(i) $n = 2$ is possible only if *the distribution of preferences is such that*

$$\begin{aligned} \max_{i=L,H} \mu_i &< \tfrac{1}{2} \quad \textit{and} \\ \max_{i=L,H} \mu_i - \tfrac{1}{3} &> \mu_M \end{aligned} \tag{1}$$

(ii) There exists $\underline{\pi}$ such that, for every $\pi > \underline{\pi}$, (1) is also sufficient.

Proof.

(I) Assume $\sigma^* = \sigma_0$ and consider first the set D_1 of distributions of preferences such that $\mu_i - \frac{1}{3} < \mu_{i'}$ for $i = L, H, i' \neq i$. In these cases the policy outcome is always t_M. By construction, then, λ_M has no incentive to accept any offer when the distribution of preferences is in D_1.

Consider now the set D_2 of distributions of preferences such that $\mu_L = \max_i \mu_i < \frac{1}{2}$ and $\mu_L - \frac{1}{3} > \mu_i$ for some i. In these cases party A_L obtains the majority of seats if $\sigma = \sigma_0$ and $Y = P$. If no politician of type H becomes a candidate at stage 2, however, the Hare quota guarantees that the median candidates grab the majority of seats. But then, if $\pi - c < t_M$ the politicians of type H receive more utility from changing the policy outcome from $t_L = 0$ to t_M (by not running) than from a seat, hence indeed decide not to run in any continuation equilibrium of σ_0. Hence no incentive once again for λ_M to accept offers.

When $\mu_H = \max_i \mu_i < \frac{1}{2}$ and $\mu_H - \frac{1}{3} > \mu_i$ for some i – call this set of distributions of preferences $D_{2'}$ – the incentive argument just made for the distributions in D_2 applies to the type L politicians *a fortiori*, since $1 - t_M > t_M$.

Finally, for any distribution in the set D_3 such that $\mu_i > \frac{1}{2}$ for some extreme party i and $\mu_i - \frac{1}{3} > \min_{i'} \mu_{i'}$, such a party obtains two seats no matter what the others do at any stage, hence, once again, no incentives to accept offers to form heterogeneous parties.

(II) Let $\pi - c > t_M$.

(i) I need to show that whenever (1) does not hold $\sigma^* = \sigma_0$. First of all, if the first inequality in (1) is the only one to be reversed, then we are in D_3, and heterogeneous parties are useless no matter what π is. Second, if the second inequality of (1) is reversed, then, regardless of what happens to the first inequality, there are two subcases: (A) $\max_{i=L,H} \mu_i - \frac{1}{3} < \mu_{i'}$ $\forall i'$; (B) $\max_{i=L,H} \mu_i - \frac{1}{3} > \min_{i=L,H} \mu_i$. Subcase (A) falls in D_1, where we know that heterogeneous parties will never be formed; Subcase (B) falls in D_2 or $D_{2'}$. In this subcase one of the extreme parties (A_H in D_2 and A_L in $D_{2'}$) would not get any seat even if $\sigma = \sigma_0$ and $Y = P$, so π can be as high as you want and would never matter: the politicians of that type would still decide not to run, hence, anticipating that, no incentives for λ_M to accept any offers.

(ii) Let $\underline{\pi}$ be the value of π such that $\pi/3 - c = t_M$. Consider first the values of $\pi > \underline{\pi}$. Let $\mu_H = \min_{i=L,H} \mu_i$. When (1) holds, party A_H must be able to obtain a seat if $\sigma = \sigma_0$ and

$Y = P$.[29] Hence if $\pi > \underline{\pi}$ the politicians of type H would run, and this creates the incentive for λ_M to accept some offers and to form a heterogeneous party. Given that the party leaders maximize the utility of the citizens of their type, λ_H has indeed an incentive to make an offer. So $n = 2$ for every equilibrium for every distribution of preferences satisfying (1).[30] However, (1) is not a sufficient condition for intermediate values of π such that $\pi - c > t_M$ but $\pi/3 - c < t_M$, since for these values there are individual incentives to run only if the seat can be obtained without having a candidate in every district. **QED.**

Proposition 2 shows that when the private benefits from holding office are sufficiently low with respect to the policy gains that can be obtained by a politician of any extreme party by not running, there is no incentive to form heterogeneous parties. The reason is that in this case the median type politicians expect that if the outcome is t_L when everybody runs then type H politicians will strategically decide not to run. On the other hand, when the private benefits from holding office are sufficiently high, no such strategic incentive can be expected, hence the median type has to accept to form a heterogeneous party in order to obtain the median outcome.

I will now show that under strategic voting the value of π loses its relevance.

Proposition 3 *The equilibria of Γ_z^{PR} have $n = 2$ if and only if (2) and (3) hold:*

$$\begin{array}{rcl} \max_i \mu_i = \mu_H & < & \frac{1}{2} \quad \textit{and} \\ \mu_H - \frac{1}{3} & > & \mu_i \ \ \textit{for some } i; \end{array} \tag{2}$$

$$t(z^*(Y,d)) = t_L \textit{ for some } Y \in \{Y : F_H(z_s(Y,d)) \geq 2\}. \tag{3}$$

Proof. I first show that whenever (2) does not hold $\sigma^* = \sigma_0$ in every SPE of Γ_z^{PR}. If $\mu_H > \frac{1}{2}$ and $\mu_H - \frac{1}{3} > \mu_i$ for some i, then we are in D_3, as defined in the proof of Proposition 2. For these distributions of preferences there is no way to take away the majority of seats to party A_H, hence (1) every continuation equilibrium of σ_0 has the same policy outcome and (2) sincere voting is Nash. Therefore heterogeneous parties are useless. If $\mu_H - \frac{1}{3} < \mu_i$ $\forall i$,

[29] To see that this must be the case, note that given (1) the shares of votes for A_L and A_M together when $Y = P$ cannot sum to more than $\frac{2}{3}$, and the remaining $\frac{1}{3}$ (or more) for A_H must be greater than μ_M, so A_H must get a seat.

[30] If (1) holds but $\max_{i=L,H} \mu_i = \mu_H$, an identical argument goes through. The only difference is that for that case the relevant lower bound is $\underline{\pi}$ such that $\pi/3 - c = 1 - t_M$.

then, regardless of what happens to the first inequality of (2), we are in D_1, as defined in the proof of Proposition 2. Hence the voters of type M expect t_M as outcome if they vote sincerely. No other type of voters has a positive benefit from voting strategically either. Thus the sincere voting profile is Nash and no other equilibrium outcome is possible when $\sigma = \sigma_0$. Hence heterogeneous parties will never be formed.[31]

Having shown the necessity of (2), let me now show that (2) and (3) together are sufficient to induce $n = 2$. Suppose that the distribution of preferences is such that (2) holds. Then if $\sigma = \sigma_0$ sincere voting is not rational, and in particular it is not Nash when $Y = P$, since the voters of type $L(M)$ could profitably deviate by voting for candidates of type $M(L)$. So, if $\sigma = \sigma_0$ and $Y = P$ there are two continuation equilibria: (1) Voters of type L vote for candidates of type M and everybody else votes sincerely (inducing $t(z^*) = t_M$); (2) Voters of type M vote for candidates of type L (at least in some districts) and everybody else vote sincerely, inducing $t(z^*) = t_L$. if the expected continuation equilibrium is (2) (i.e., if (3) holds), then λ_H would have incentive to deviate and offer λ_M to form a party, with an offer that λ_M would accept. The only equilibria of Γ_z^{PR} that are compatible with (2) have both extreme parties compete to have λ_M accept the offer, hence the party structure has a heterogeneous party. The necessity of (3) can easily be understood by noting that if (3) is violated then it must be the case that the continuation equilibrium of $Y = P$ is (1); then, like with sincere voting, no incentive at stage 1 to form a heterogeneous party. **QED.**

Proposition 4 *In Γ_z^{PR} a strong equilibrium always exists, and the unique equilibrium party structure in a strong equilibrium is $\sigma^* = \sigma_0$, for all distributions of preferences.*

Proof. As shown in the above proof, at all subgames where sincere voting is not Nash, i.e., $\forall Y : z_s(Y, d) \notin Z^*(Y, d)$, there are two types of continuation equilibria. The continuation voting equilibria such that $t(z^*(Y, d)) = t_M$ (with $z_L^l(P^l) = M$ and everybody else vote sincerely in enough districts) are clearly robust to coalitional deviations at the voting stage. Hence a strong equilibrium always exists, inducing $\sigma^* = \sigma_0$. To see that such a party structure is the unique one compatible with the strong equilibrium, note that if another Nash equilibrium with $z_M^{l*}(P^l) = L$ in some district l is the continuation voting profile,

[31] Note that when (2) holds with L instead of H on the left hand side, the voters of the median party always vote sincerely because the other small party they could vote for in order to defeat the relative majority party has the more distant policy position. Hence no incentive to form heterogeneous parties.

causing $t(z^*) = t_L$, then the voters of type M and H can profitably deviate by following a deviating recommendation by their party leaders to vote for type M candidates if $Y^l = P^l$. **QED.**

Proposition 3 highlights an important difference between Γ_s^{PR} and Γ_z^{PR}. Under sincere voting the equilibrium party structure depends on the distribution of preferences and on the relative size of private benefits and policy gains by not running, whereas under strategic voting only the expectations of voters' behavior matter. Moreover, if voting recommendations must not only be Nash but also immune from coalitional deviations, then Proposition 4 shows that no heterogeneous party ever forms. The fact that $n = 3$ in a strong equilibrium of Γ_z^{PR} does not mean anything for an evaluation of Duverger's hypothesis, because not all the three parties need to be active or effective, as I will show in the next section.

The three propositions above fully characterize the equilibrium party structure under PR for every equilibrium of the games with and without strategic voting, and for every distribution of policy preferences. It is now possible to characterize the equilibrium policies.

Proposition 5 *The equilibrium policy outcome under Proportional Representation is t_M unless*

$$\begin{array}{rcl} \max_{i=L,H} \mu_i & > & \frac{1}{2} \\ \max_{i=L,H} \mu_i - \frac{1}{3} & > & \min_{i'} \mu_{i'}. \end{array} \tag{4}$$

Proof. As under PV, the characterization of equilibrium policy is *not affected* by the voting assumption. Both (1) and (2) are violated when (4) holds, so it is clear that when (4) holds there is never any heterogeneous party and the policy is determined by the absolute majority party. In other words, (4) is clearly sufficient to determine an outcome different from t_M. In order to establish necessity, note that when the second inequality of (4) is reversed then each party gets a seat in every equilibrium of both games, and the outcome is therefore t_M; If the first inequality is violated, then the necessary conditions for $n = 2$ may be satisfied, but in equilibrium the policy compromise of a heterogeneous party is t_M anyway. This is because the offer stage is competitive, and hence in any equilibrium with $n = 2$ both extreme parties must offer $\tau^* = t_M$. **QED.**

4 Comparative Results

Given the complete characterization of equilibria provided in the previous section, I am now able to derive, highlight and discuss the main comparative results of the paper.

4.1 Duvergerian Predictions

Recall the definitions of the three Duvergerian predictions (i.e., Duverger's law, Duverger's hypothesis, and Duvergerian comparative prediction) from the first paragraph of the introduction. In this section I am going to highlight the implications of the characterization results just obtained for all of them, in that order.

Remark 1 and Proposition 1 imply the following corollary result:

Corollary 2 *Under Plurality Voting there are three active (and effective) parties if and only if the distribution of preferences is in the set D_b, which denotes the set of all distributions such that $\mu_H^l > \frac{1}{2}$ in district l, $\mu_L^{l'} > \frac{1}{2}$ in district l', and $\mu_i^{l''} < \frac{1}{2}$ in district l'', $i = L, H$.*[32]

This constitutes a realistic qualification of Duverger's Law in contexts of multiple districts. It says that Duverger's law extends (i.e., there are at most two effective parties in the polity) unless the districts are so heterogeneous that each party contains the median voter of one district. It is realistic because it combines the intuition of Duverger's law for a unified electorate with the observation that heterogeneous countries like India continue to have many active and effective parties even though they use PV.

Under PR it should be clear that if the distribution of preferences is somewhat concentrated, as summarized in the following corollary, then the number of active and effective parties *must* be less than 3.

Corollary 3 *If $d \in D_c \equiv \{d : \mu_i > \frac{1}{2}$ and $\mu_i - \frac{1}{3} > \mu_{i'}$ for some $i, i'\}$, then there cannot be three active (nor effective) parties in any equilibrium under PR.*

Note that $d \in D_c$ is sufficient but *not necessary* to have a reduction of effective parties. To see this, recall from Proposition 2 that when private benefits from being elected are very high there are strong incentives to party formation in the subset of $D \setminus D_c$ that satisfies (1);

[32]For active parties, this ignores the non-generic case mentioned in footnote 28.

also recall that under strategic voting there may be incentives to party formation in $D \setminus D_c$ if the expected continuation voting equilibrium is not the strong one; Finally, even when limiting attention to strong equilibrium, there can still be less than three active and effective parties in $D \setminus D_c$ when the relative majority party (in terms of preferences) is the median party.[33] Since the incentives to party formation vary across these cases, it is not possible to provide the reader with a unified and tight necessary condition to have less than three effective parties in PR, but it should be clear that it is far from unlikely.

It is striking to notice that whenever multipartyism (predicted by Duverger's hypothesis) is not observed,[34] the third party has always something very close to 30% less than the majority party in the polls. This suggests that those "exceptions" to Duverger's hypothesis in national elections are *not* anomalies, and should not be surprising given the distribution of policy preferences.

So far I have clarified the sense in which Duverger's law and Duverger's hypothesis depend on the distribution of preferences being not too different across districts and not too concentrated or polarized within districts. The comparison of Corollary 2 and Corollary 3 can now help to check whether even the Duvergerian comparative prediction can be reversed.

It is clear that when the distribution of preferences is sufficiently close to uniform in all districts, then the Duvergerian comparative prediction extends to multi-district elections. For any distribution of preferences in the neighborhood of $d : \mu_i^l = \frac{1}{3}\ \forall i, \forall l$, there is only one active and effective party with Plurality and three active and effective parties under PR. However, if the distribution of preferences is sufficiently heterogeneous, things may go the opposite way, as in the following example:

Example 1 *Duvergerian predictions can be reversed.*

	north	*center*	*south*	*total*
left-wing	*0.6*	*0*	*0*	*0.2*
right-wing	*0*	*0*	*0.63*	*0.21*
moderate	*0.4*	*1*	*0.37*	*0.59*

[33] For example, if the median party has 46% of the preferences and the others have 42% and 12%, then only the first two parties are active and effective in equilibrium.

[34] See the discussion on Australia,Ireland and Germany, for example, in Lijphart (2001).

The last column represents μ_i for every party i; the generic element of the rest of the matrix has a μ_i^l. It is easy to check that sincere voting is rational in this example, and hence coincides with the strong equilibrium. So the distribution of preferences in the matrix translates one-to-one in a distribution of votes, and the policy outcome is the same with the two electoral systems. There are three active and effective parties with Plurality, whereas there are only two under PR, so *the Duvergerian comparative prediction is reversed.* Under PV each party has the majority of preferences in one district, and extremist politicians decide to run where they can win even if they will not affect the policy outcome, because they like to be elected. Under PR the politicians of the right-wing party expect that one of them will be elected if they all run; so they run, given $\pi/3 > c$. Consequently, no politician from party L decide to run because they know that they would not be elected and there is a cost of running.

Recalling the definitions of D_b and D_c in the above corollaries, every $d \in D_b \cap D_c$ displays the reversal of the Duvergerian comparative prediction illustrated in this example. But $d \in D_b \cap D_c$ is only a sufficient condition, for the reasons discussed following Corollary 3. Intuitively the reversal should become more likely in a model with more types and more districts, since the asymmetries between districts and the concentration of preferences within districts don't need to be that sharp.

A final remark is that because of complete information the equilibrium under PV displays a unique runner in every district, rather than two as you would most likely have with incomplete information about voters' preferences or with probabilistic voting. Since the Duvergerian comparative prediction can be reversed in spite of the unique-runner feature of complete information, it would seem *a fortiori* all the more plausible that this possibility should extend to a world with incomplete information.

4.2 Welfare Analysis

The choice of an electoral system may have welfare consequences. In particular, I will now show that, in spite of the uni-dimensional policy space and the linear utility functions, there are distributions of policy preferences such that PR is welfare superior to PV, and vice versa. I will then argue that the parameter region where PV dominates PR vanishes when the utility functions are made sufficiently concave.

Given the utility functions assumed in this paper, an electoral system maximizes welfare if its induced policy outcome coincides with the median voter's preferred policy. Under PR, having the absolute majority of preferences is not a sufficient condition to obtain two seats. For example, an extreme party with 52% of the preferences obtains only one seat if the other two parties have 24% each. Hence the policy outcome is t_M, which is not the median voter's preferred outcome in this case. The relationship between the policy outcome under PR and the median voter's preferred policy is characterized by the following remark:

Remark 2 *The equilibrium policy outcome of* Γ_s^{PR} *and* Γ_z^{PR} *is the one preferred by the median voter unless the following two conditions hold:* $\max_{i=L,H} \mu_i > \frac{1}{2}$ *and* $\max_{i=L,H} \mu_i - \frac{1}{3} < \mu_{i'} \quad \forall i'$.

Under PV, given Corollary 1, the set of parameters generating an outcome different from the median voter's preferred policy is characterized as follows:

Remark 3 *The equilibrium policy outcome of* Γ_s^{PV} *and* Γ_z^{PV} *is the one preferred by the median voter unless*

either

(1) $\mu_i^l > \frac{1}{2}$ *in two districts for some* $i \in \{L, H\}$, *but* $\mu_i < \frac{1}{2}$;

or

(2) $\mu_i > \frac{1}{2}$ *for some* $i \in \{L, H\}$ *but* $\mu_i^l > \frac{1}{2}$ *in only one district.*

Under PR it may happen that the policy outcome is moderate even when the median voter is actually of one of the two extreme types. The opposite cannot happen. On the other hand, with PV it may happen that the policy outcome is one extreme even though the median voter is of the median type or even of the opposite extreme type. So PR always yields moderate outcomes (sometimes too moderate), whereas under PV the majority of preferences in the country is irrelevant and hence the policy outcome can be anywhere with respect to what the median voter wants.

The following two examples show a case in which PR dominates PV (example 2) and one in which PV dominates PR (example 3).

Example 2

	north	*center*	*south*	*total*
left-wing	*0.51*	*0.51*	*0.24*	*0.42*
right-wing	*0.2*	*0.2*	*0.23*	*0.21*
moderate	*0.29*	*0.29*	*0.53*	*0.37*

PV: t_L*; PR and optimal:* t_M*.*

Example 3

	north	*center*	*south*	*total*
left-wing	*0.51*	*0.51*	*0.51*	*0.51*
right-wing	*0.2*	*0.2*	*0.32*	*0.24*
moderate	*0.29*	*0.29*	*0.17*	*0.25*

PR: t_M*; PV and optimal:* t_L*.*

Note that to go from the preferences in example 2 (where PR dominates PV) to those in example 3 (where PV dominates PR) a shock to the policy preferences in the south is sufficient. Both systems can be inefficient, but there are two considerations that must be made in "favor" of PR: First of all, if the representative democracy game presented here is played every election period, and if policy preferences are subject to random shocks from period to period, then a small amount of risk aversion would make citizens prefer PR, since it is always (weakly) more moderate than PV. PV should be expected to determine more variance in terms of policies over time.

The second consideration has also to do with introducing some more concavity in the utility function: assume, for instance, that the loss function for a citizen of type i is $(t_i - t^*)^2$; then, assuming that t_M is close enough to $\frac{1}{2}$, in example 3 the welfare ranking switches, and PR dominates PV there too. More generally, having a bias towards the center, with respect to what the median voter wants, becomes a plus (rather than a minus) the more concave is the utility function.[35]

4.3 Sincere vs. Strategic Voting

Economic theory and formal political theory do not have a solid explanation (yet?) about *why* people vote, and this paper does not attempt to solve this problem either. Assuming

[35]See Maskin (1979) for a clear discussion of how the welfare analysis of majority rule depends on the curvature of the utility function.

any level of participation in an election, the next question is *how* do they vote. Do they vote sincerely or are they willing to vote for their second or third "closest" candidate if this way they can obtain a more desirable policy outcome? In this section I show that in more than one sense *it does not matter*. The next two remarks highlight what we can learn about this issue from Propositions 1 and 3; the subsequent discussion will make clear in what sense the voting assumption is irrelevant.

Remark 4 *Even though sincere voting strategies are not necessarily rational, the equilibrium voting behavior under PV is* always sincere*. In fact, $Y^l = P^l$ cannot happen in any district in any equilibrium when candidacy is endogenous.*

Remark 5 *Sincere voting is rational under Proportional Representation if and only if (2) does not hold. Moreover, when (2) holds there are always some voters who do not vote sincerely in equilibrium.*

Comparing Remarks 4 and 5, one should note that it is important to distinguish between strategies and actions: if one asks whether sincere voting is rational or not, the question is in terms of strategies, and the answer is that sincere voting is more often rational under PR; on the other hand, if one asks whether the actual equilibrium behavior is sincere, the question is in terms of actions, and in this case the answer is reversed: the equilibrium voting behavior is always sincere under PV, whereas under PR if (2) holds there are always voters who do not vote sincerely in equilibrium. In other words, only under PR we could *observe* strategic voting behavior.

The intuitive reason for this sharp contrast is that even though under PV there are always reasons to be strategic when $Y^l = P^l$, these subgames are never reached in equilibrium; on the other hand, under PR it is very common that all the potential candidates run (one for every party in every district), and hence, whenever there are reasons to be strategic, strategic voting is observed in equilibrium. This very same intuition can be invoked to explain the irrelevance of the voting assumption under PV, as shown in Proposition 1, and to explain the irrelevance of the voting assumption for the determination of policy outcomes under both electoral systems. The following remark highlights this irrelevance:

Remark 6 *Given the possibility of party formation and endogenous candidacy, the equilibrium policy outcome is not affected by whether voters are expected to be sincere or strategic. Thus, the sincere vs. strategic voting issue is* irrelevant *for welfare analysis.*

The voting assumption has some effects on the equilibrium party structure though, in the sense that under sincere voting party formation is an equilibrium phenomenon under PR for sufficiently high private benefits from election, whereas when voters are expected to follow strong voting recommendations no party formation occurs. This fact that sincere voting can induce *more* party formation than strategic voting can also be observed for PV, but only in the special case where candidacy is not endogenous, as argued in the next section.

5 Robustness, Extensions, and Generalizations

The results of this paper are robust, in the sense that altering the model in all possible reasonable ways one does not find any significant impact on the comparative analysis. I will first discuss what happens when candidates are chosen by the parties (closed lists), suggesting some additional institutional comparisons. Second, I will argue that stage 1 of the game can easily be made more general, endogenizing the credibility of policy compromises. Third, I will show that the order of play at stage 2 is substantially irrelevant. Other extensions and generalizations will be mentioned (and are available upon request).

5.1 Closed Lists

Let us consider a simplification of the game, in which voters vote for parties directly (an extreme form of closed list). In other words, consider the games G_s^E and G_z^E that are like Γ_s^E and Γ_z^E (respectively) but dropping stage 2 altogether and letting voters choose one of the endogenous parties.

5.1.1 Plurality Voting

In contrast with Proposition 1 (in which the voting assumption plays no role), when voters vote for parties and candidates are not endogenous the equilibrium party structure *is* affected by how citizens are expected to vote. The following proposition formalizes this claim.

Proposition 6 *In game G_s^{PV} $n = 2$ if and only if $d \in D \setminus D_a$ and an extreme type has the* relative *majority of preferences in at least two districts.*

On the other hand, in G_z^{PV} $n = 3$ in a strong equilibrium.

The proof is omitted. The intuition can be given with a simple example. Suppose the three types have roughly equal numbers of supporters in every district (μ_i^l in a neighborhood of $\frac{1}{3}$ for every i and every l), but let $\max_i \mu_i^l = \mu_L^l$ in at least two districts. With sincere voting party A_L would win the majority of seats if $\sigma = \sigma_0$, because there is no stage 2; hence λ_H has an incentive to offer a policy compromise to λ_M, and in equilibrium a heterogeneous party will form. On the other hand, if the strong equilibrium is expected for any party structure, no incentives to party formation exist. The irrelevance of the voting assumption continues to hold in terms of policy outcome.

5.1.2 Proportional Representation

When voters vote for parties and are sincere, Proposition 2 is replaced by the following:

Proposition 7 *If voters vote for parties and are sincere, then under Proportional Representation $n = 2$ if and only if*

$$\frac{1}{2} > \max_i \mu_i = \max_{i \in \{L,H\}} \mu_i > \min_i \mu_i + \frac{1}{3}. \tag{5}$$

Whenever (5) holds, the second and third parties have incentive to form an electoral coalition before the elections, and for the usual reasons the heterogeneous party has t_M as policy compromise. Necessity can also be easily established.

Propositions 3 to 5 extend without modification. The reason is that, as shown in Section 3.2, candidates' motivations are an important variable only with sincere voting.

In summary, without endogenous candidacy party formation becomes more likely an equilibrium phenomenon under both systems when voters are sincere. The irrelevance in terms of policy outcomes suggests instead that the choice between open and closed list has no welfare consequence.

5.2 Endogenous Credibility

An assumption of the model is that policy compromises agreed upon in a heterogeneous party are perfectly credible to voters.[36] This credibility property can actually be obtained endogenously. Note that the only possible equilibrium policy compromise is $\tau = t_M$. This implies that I could have assumed that at stage 1 the extreme party leaders can simply propose to withdraw their politicians, rather than proposing a policy compromise. The extreme party leaders would be willing to make the withdrawal proposals in the same circumstances where they propose a compromise in the current model, and the results would therefore be absolutely identical, with no credibility assumption.

In the current formulation of stage 1 a heterogeneous party $A_L \cup A_H$ can never form. This could be made an equilibrium phenomenon with the same modification discussed in the previous paragraph: $A_L \cup A_H$ could not obtain a credible compromise, since all the politicians of that heterogeneous party would be extreme. Thus, with the modification proposed above stage 1 can be made a more symmetric game where every coalition is feasible, and it is easy to show that all the results of the paper extend (calculations available upon request). The fact that in all the generalizations of the model (that I deem reasonable) the equilibrium heterogeneous parties (if any) are always "connected" coalitions (i.e., only coalitions of adjacent types) is not surprising, and is consistent with real world observations.[37]

5.3 Random Order of Running Decisions

One could ask whether the results of this paper are robust to changes in the order of play at stage 2. The answer is yes, at least in terms of the comparative issues highlighted in Section 4. Even the characterization results are very similar. In particular (proof available upon request) it can be shown that:

Remark 7 *The results for* Γ_z^{PV} *remain true* for every *order of play at stage 2;*
The results for Γ_s^{PV} *remain true for every order of play at stage 2 in terms of policy outcome,*

[36]To see different mechanisms that parties could use to make moderate platforms credible in various contexts, and to see how that affects policy convergence in electoral competition models with two parties, see Alesina and Spear (1988) and Harrington (1992).

[37]To see other intuitive justifications for considering only coalitions of adjacent types in general, see Axelrod (1970).

but there is one case in which $n = 2$, namely when the order of play is L, M, H.

Under PR the generalization remark would be along the same lines. Propositions 3 to 5 can be shown to extend word by word, since the order of play at stage 2 never enters the arguments in their proofs. Like for PV, it is only with sincere voting that the order of play may change the equilibrium characterization. When the type L politicians move first they may decide to run even when π is small. Then a heterogeneous party has to form in anticipation, regardless of π.

If the order of play is random the party formation decisions are made with some probability distribution in mind about the order of running decisions, but the additional incentive to form heterogeneous parties under sincere voting arises as soon as the order L, M, H has positive probability. Thus, like in Section 5.1, all that might change with respect to the results in the paper is in terms of a stronger effect of sincere voting on party formation.

5.4 Other Generalizations

A generalization that is worth discussing concerns the number of policy preferences, or types. The only modeling change needed to allow for more than 3 types is at stage 1. Each party leader would have to make a proposal concerning (1) a heterogeneous party, (2) a policy compromise within that proposed coalition, and (3) a list of candidates in order to make the compromise credible. Then, as mentioned in Section 5.2 for the three-type case, all the results would extend (and only connected coalitions would form). One potentially interesting aspect of enlarging the set of types is that under PV one could obtain two running candidates in some district in spite of still having complete information. To see this, imagine to have four types, with the two in the middle being relatively close to each other; then there would be symmetric distributions of preferences (giving to the left-most two types half of the citizens' preferences) such that in equilibrium two parties would form and, in at least some districts, each of the two parties would have a candidate.[38]

A set of interesting future generalizations will involve either assuming that politicians have to make decisions only using an "expected" distribution of policy preferences, or assuming that voters only know the expected position of politicians, or assuming both types

[38] In earlier versions of this paper I had a more complicated model with general number of types, seats, and districts, so some of the generalizations of this kind are available upon request.

of incomplete information. In particular, Myatt (1999) has shown that if public signals are not very informative then strategic voting is not very effective, and hence I expect that introducing this type of incomplete information in my model I will be able to show that the less informative are public signals the more important become strategic party formation and strategic candidacy.

Last but not least, it will be interesting (but very challenging) to study the extension of the model to a multidimensional policy space. The after-election parliamentary stage will have to be modeled with more institutional assumptions, since pure majority rule would often lead to cycles. The role of parties during the elections will probably be the hardest to characterize, whereas the role of parties as commitment device will likely be enhanced, as suggested in Levy (2000). To see how the after-election bargaining game could be modeled in the presence of multiple policy dimensions, see Morelli (1999).

6 Concluding Remarks

The first authors to study representative democracy with endogenous candidates are Osborne and Slivinsky (1996) and Besley and Coate (1997). Osborne and Slivinski only have sincere voting, whereas in Besley and Coate's model citizens are strategic both at the candidacy stage and when they have to vote. In these models parties are missing and there is only one district. In contrast, in this paper parties play a role both *ex ante*, as commitment devices for politicians, and during the elections, where they coordinate voters' behavior. With this important addition the model makes a unique prediction in terms of policy outcome and party structure for every distribution of policy preferences, whereas in Besley and Coate the multiplicity of equilibrium outcomes is unavoidable, and this makes comparisons of electoral systems hard. The importance of conducting a comparison of electoral systems in the presence of endogenous candidacy can be stressed by referring the reader to the recent paper of Dutta, Jackson, and Le Breton (2001): They show that for a comparative analysis of voting procedures it is often necessary to take into account the endogenous incentives to run, because most well known voting mechanisms are not candidate stable.

As pointed out in the introduction, the formal models on Duverger's law all focused on strategic voting with fixed candidates in a unified district.[39] This paper has extended the analysis to multi-seat/multi-district national elections with parties connecting the candidates of the various districts, and all the Duvergerian predictions have been qualified and checked for every distribution of preferences. One important contrast between this paper and the literature just cited on Duverger's law is the following: with fixed candidates in a unified district the force leading to a reduction of parties is strategic voting; on the other hand, in a world with multiple districts and endogenous parties there are stronger forces leading to a reduction of the number of parties if voters are expected to be sincere. The reason for this contrast is that other strategic players (parties and/or candidates) can *substitute* strategic voting for the task of obtaining a Nash equilibrium behavior.

The Duvergerian predictions have been shown to hold in a representative democracy with multiple districts as long as the distribution of policy preferences is close enough to uniform within districts and not too dissimilar across districts. On the other hand, if the distribution of policy preferences is somewhat asymmetric, then *all* the Duvergerian predictions can be *reversed*.[40] The important role of policy preferences has been emphasized also in terms of their implications for the welfare analysis of the two electoral systems. Under some distributions of policy preferences the choice of an electoral system is irrelevant for the policy outcome, since the median voter's preferred outcome is the equilibrium outcome of both Proportional Representation and Plurality Voting. However, there is a positive measure of distributions of policy preferences such that the policy outcome differs from the median voter's preferred one. Under Plurality Voting the policy outcome may be more "extreme" than the optimal one, namely when an extreme party has the absolute majority of preferences in the majority of districts but not in the whole country. It can also be more "centrist" than what the median voter wants, when an extreme party has the absolute majority of preferences in the whole country but the absolute majority in a minority of districts. On the other hand, under Proportional Representation the policy outcome can only be more "centrist" than the optimal policy, never more "extreme". So the two electoral systems may determine suboptimal outcomes, different from what the median voter wants, but if they do, they often

[39]See Cox (1997), Fey (1997), Feddersen (1992), and Palfrey (1989).

[40]For a discussion of other political effects of electoral systems that could be checked using the model presented in this paper in future research, see Taagepera and Shugart (1989), Lijphart (2000) and Myerson (1999).

do so in opposite directions. Note that this welfare analysis is not affected at all by whether voters are expected to be sincere or strategic.

The model is probably too stylized to give very precise empirically testable predictions, but one speculation that I would venture to make is the following: Over time policy preferences change and swing, so departures from the median voter's theorem are likely in the long run; since this model suggests that the deviations from the median voter's preferred policy are always in the moderate direction for PR but in all directions under PV, I would expect the *variance* of policy outcomes to be higher in a time series for a country using PV than in a time series for a country using PR. This conjecture speaks in favor of PR, especially if the utility functions of citizens are concave.

References

[1] ALESINA, A. (1988), "Credibility and Policy Convergence in a Two-Party System with Rational Voters," *American Economic Review*, **78**-4, 796-805.

[2] ALESINA, A. AND S. SPEAR (1988), "An Overlapping Generations Model of Electoral Competition," *Journal of Public Economics*, **37**, 359-79.

[3] ALESINA, A. and R. ROSENTHAL (1996), "A Theory of Divided Government," *Econometrica*, **64**, 1311-41.

[4] AXELROD, R.M. (1970), *Conflict of Interest; A Theory of Divergent Goals with Application to Politics*, Chicago, Markham.

[5] BARON, D. (1993), "Government Formation and Endogenous Parties," *American Political Science Review* **87**, 34-47.

[6] BESLEY, T. and S. COATE (1997), "An Economic Model of Representative Democracy," *Quarterly Journal of Economics* **112**, 85-114.

[7] CAILLAUD, B. and J. TIROLE (1999), "Party Governance and Ideological Bias," *European Economic Review,* **43**(4-6), 779-89.

[8] CHHIBBER, P. and K. KOLLMAN (1998), "Party Aggregation and the Number of Parties in India and the United States," *American Political Science Review*, **92**(2), 329-42.

[9] COX, G.W. (1997), *Making Votes Count: Strategic Coordination in the World's Electoral Systems,* Political Economy of Institutions and Decisions series, Cambridge; New York and Melbourne: Cambridge University Press.

[10] DE SINOPOLI, F. and G. IANNANTUONI (2001), "A Spatial Voting Model where Proportional Rule Leads to Two-Party Equilibria," Mimeo, University of Rochester.

[11] DUTTA, B., M.O. JACKSON, and M. LE BRETON (2001), "Candidacy and Voting Procedures," *Econometrica*, **69**(4), 1013-37.

[12] DUVERGER M. (1954), *Political Parties,* Wiley, New York.

[13] FEDDERSEN, T.J. (1992), "A Voting Model Implying Duverger's Law and Positive Turnout," *American journal of political science,* **36**-4, 938-62.

[14] FEY, M. (1997), "Stability and Coordination in Duverger's Law: A Formal Model of Preelection Polls and Strategic Voting," *American Political Science Review,* **91**, 135-47.

[15] HARRINGTON, J. (1992), "The Role of Party Reputation in the Formation of Policy," *Journal of Public Economics*, **49**, 107-21.

[16] JACKSON, M. and B. MOSELLE (2001), "Coalition and Party Formation in a Legislative Voting Game," *Journal of Economic Theory*, forthcoming.

[17] LEVY, G. (2000), "The Role of Parties and Endogenous Parties in Multidimensional Policy Space," mimeo, London School of Economics.

[18] LEYS, C. (1959), "Models, Theories and Theory of Political Parties", *Political Studies*, **7**, 127-46.

[19] LIJPHART, A. (2000), *electoral Systems and Party Systems*, Oxford University Press.

[20] LIZZERI, A. and N. PERSICO (2001), "The Provision of Public Goods under Alternative Electoral Incentives," *American Economic Review*, **91**(1), 225-39.

[21] MASKIN, E. (1979), "Voting for Public Alternatives: Some Notes on Majority Rule," Proceedings of a Conference on Tax and Expenditure Limitations, *National Tax Journal*, **32-2**, 105-9.

[22] MORELLI, M. (1999), "Demand Competition and Policy Compromise in Legislative Bargaining," *American Political Science Review*, **93-4**, 809-20.

[23] MYATT, D.P. (1999), "A New Theory of Strategic Voting", mimeo, Nuffield College, Oxford.

[24] MYERSON, R. (1999), "Theoretical Comparisons of Electoral Systems," *European Economic Review*, **43**(4- 6), 671-97.

[25] OSBORNE, M. and A. SLIVINSKY (1996), "A Model of Political Competition with Citizen Candidates," *Quarterly Journal of Economics,* **111**, 65-96.

[26] PALFREY, T.R. (1989), "A Mathematical Proof of Duverger's Law," in: Ordeshook, P.C. (ed.), *Models of strategic choice in politics,* Ann Arbor: University of Michigan Press. Pp. 69-91.

[27] PERSSON, T. and G. TABELLINI (1999), "The Size and Scope of Governments. Comparative Politics with Rational Politicians. 1998 Marshall Lecture," *European Economic Review*, **43**, 699-735.

[28] RIKER, W. (1982), "The Two-party System and Duverger's Law," *American Political Science Review*, **76**, 753-66.

[29] RIVIERE, A. (1998), "Citizen Candidacy, Party Formation, and Duverger's Law," mimeo ULB.

[30] SARTORI, G. (1967), *Political Development and Political Engineering*, Bloomington Indiana, Indiana University.

[31] TAAGEPERA, R. and M.S. SHUGART (1989), *Seats and Votes: The Effects and Determinants of Electoral Systems,* Yale University Press, New Haven, Connecticut.

[32] WITTMAN, D.A. (1977), "Candidates with Policy Preferences: Dynamic Model," *Journal of Economic Theory*, **14**-1, 180-89.