

Market Power and Information Revelation in Dynamic Trading*

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Abstract

We study a strategic model of dynamic trading where agents are asymmetrically informed over common value sources of uncertainty. There is a continuum of uninformed buyers and a finite number of sellers, some of them informed. When there is only one seller, full information revelation never occurs in equilibrium and the only information transmission happens in the first period. The outcome with n sellers depends both on the structure of sellers' information and, more importantly, on the intensity of competition among them allowed by the trading rules. With intense competition (absence of clienteles), information is fully and immediately revealed to the buyers in every equilibrium for n large enough, both when all sellers are informed and when only one seller is informed. On the other hand, with a less intense form of competition (presence of clienteles), we always have equilibria where information is never fully revealed, whatever

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the number of sellers. Moreover in this case, when only one seller is informed, for many parameter configurations there are no equilibria with full information revelation, for any n .

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1. Introduction

This paper studies a strategic model of dynamic trading where: (1) non-negligible agents interact with negligible ones –a finite number of sellers serve a continuum of buyers, and (2) there is asymmetric information about the quality of the good being traded, an instance of common values uncertainty. In this situation the sources of a seller’s market power are two-fold: his size relative to the market allows him to manipulate the terms of trade, and his private information serves him to manipulate the other traders’ beliefs. Furthermore, since trade extends over infinitely-many periods, current actions have important effects on the agent’s market power in the future (if today’s action were to completely reveal the trader’s information, his informational advantage would be null in the future; similarly, a gain in today’s market share may only come at the expense of future trades).

We are interested in identifying how the agents’ market power depends on their size and the nature of their information, and the implications of this for the properties of the trading process. In the dynamic set-up we consider, we can then analyze how market power evolves over time as well as if, and how fast, private information is revealed at equilibrium. We shall identify then the conditions under which the equilibria with strategic trading “converge” to the “competitive” equilibria.

The most frequently used Walrasian equilibrium notion in economies with asymmetric information over common values sources of uncertainty is given by rational expectations equilibria (REE).¹ These are (generically) characterized by the fact that equilibrium prices fully and immediately reveal all the agents private information. Such notion is more demanding than usual Walrasian equilibria, since it requires both that agents have no market power (act as price-takers) and that all private information is disclosed.

¹See Radner (1982) for a survey.

The strategic foundations of REE require then the investigation of trading procedures where the transmission of information among the agents in the market is explicitly modelled. One such decentralized trading procedure was explored by Wolinsky (1990), Blouin and Serrano (2001), and others. They consider economies with infinitely many trading dates. Each period agents chosen from a continuum are randomly matched in pairs; within each pair they bargain over the terms of trade (rejection of all offers and hence refusal to trade is always an available option). Two aspects of this trading procedure are worth stressing. First, information is heavily restricted; agents have no access to any public market signal, nor do they have any information over the terms at which trade occurred in the past in the market. And second, each pairwise meeting (which can be thought of as a buyer visiting a particular store) represents a “local monopoly” in its own right, in the sense that in that period the buyer can buy the good only from that store. Of course, he can walk out and visit a different store, but this comes at a cost, as captured by discounting. The findings of these papers are very far from the convergence to fully revealing REE: even in the limit when discounting is zero and there are many traders, a significant fraction of them informed, a sizeable portion of uninformed traders transact at prices that are *ex post* not individually rational.

Fully centralized trading mechanisms were analyzed in various other papers. Dubey, Geanakoplos and Shubik (1987) and more recently Forges and Minelli (1997) studied the case where trading takes place via market games a la Shapley-Shubik, in the presence of a continuum of agents of finitely many types (each source of information is possessed so by a continuum of agents). They showed that, when the trading game is repeated, the Nash equilibria are such that the first stage is used to exchange information among traders (equilibrium prices act as public signals), and in the subsequent stages the outcome coincides, under appropriate conditions, with the fully revealing REE. The case in which there is a single informed trader acting strategically on the basis of his information, also with centralized trades, was considered then by Grinblatt-Ross (1985), Laffont-Maskin (1990) within a one-period trading model, and by Kyle (1985) in a dynamic trading model. In both cases it was shown that the monopolist may choose not to completely reveal his private information and that equilibria differ from REE. The result is more surprising in a dynamic set-up because hiding the information over time is a harder task. Some qualifications are however needed here. Kyle (1985) considers a model where noise traders are also present and prices are determined each period by competitive market-makers on the basis of the observation of ag-

gregate trades. Thus aggregate trades, and a fortiori prices, cannot fully reveal the private information of the informed trader. Even if he were to act non strategically, full revelation could only be achieved in the limit, after infinitely many rounds of trade. When the informed acts strategically, convergence of prices to their full information value also occurs, but it is slower since the informed “hides” his trades from the market.

In this paper we attempt to reconcile these seemingly contradictory results. We will study a highly stylized model of dynamic trading, where all traders are fully rational (there are no noise traders) and sellers compete by posting prices at which they are willing to sell. One criticism of the pairwise meetings papers mentioned above is that it has gone too far in limiting the possibilities of information transmission among traders. It is plausible to model a situation in which traders receive some market signal: in this paper we shall assume that each period buyers get to observe *all* prices posted by sellers before they agree to buy the good.² We analyze the model under various market and information structures, to identify the conditions under which information revelation occurs. In each case, we will provide a complete characterization of all the equilibria and analyze their efficiency properties. The specifics of our model follow.

We study a market for an indivisible good of uncertain quality with infinitely many trading dates. All the units of the good in each period are of the same quality, either high or low. Thus, there are two states of the world: H and L . There is a continuum of buyers, all of them uninformed about the state. There is a finite number n of sellers, at least one of them informed. Every period, each seller simultaneously chooses whether to post a high or a low price for the good.³ Upon observing the prices posted in a period, every buyer can either refuse to buy (in which case he will continue to be in the market the next period), or accept to buy from one of the sellers at the price he announced (in which case he will leave the market forever). Even though quite stylized, it may help, to fix ideas, to think of the model as portraying situations like a specific wine crop sold in a fair by the producers, or the market of derivatives on (aggregate) credit risk. In both situations we can argue all units sold are of the same quality, buyers and sellers are clearly identified, and there are reasons for trade other than differences in

²See Peters (1991) for a related model, where agents’ matching is non-random because it is affected by the posted prices.

³The restriction of having only two possible prices strips down the model of unnecessary complications (in particular, by limiting the role of the beliefs of the uninformed) and allows to provide a complete characterization of all the equilibrium outcomes. As argued in Section 3.2, the substance of our results remains valid even if this restriction is dropped.

information. More generally, one may view informational asymmetries in financial markets as being typically over common values uncertainty (e.g. over the asset returns) and in fact the main motivation of Kyle's work and the literature which followed it was the study of trading in financial markets.

We consider first the case of an informed monopolist ($n=1$). We find that at equilibrium he reveals some information only when the buyers' prior belief is too pessimistic about the state being H ; in this case he reveals only part of his information, and only in the first period. In contrast to Kyle's (1985) results, in the absence of noise traders continued randomization, which progressively reveals the information, is not optimal; once the buyers are convinced that consuming the good brings them a non-negative expected utility, the single informed trader has no further incentive to reveal his information. This section serves as an important benchmark for the analysis in the following sections.

The paper proceeds to analyze the case of oligopoly under two, rather extreme, information structures: (i) all sellers are informed (information is thus non-exclusive and each seller is "informationally small" – see Gul and Postlewaite (1992)); (ii) only one seller is informed (he is then "informationally big" relative to the other agents).⁴ We also allow for two alternative sets of trading rules, characterized by the difference in the intensity of price competition that is permitted. In the first one - denoted as the model without clienteles - each buyer is free to buy the good from the seller that is offering it at the lowest price (as in the classic Bertrand model). On the other hand, in the model with clienteles any seller can only reach within each period a fraction of all sellers (as in Cournot, or Bertrand with capacity constraints).

We show that in the model without clienteles, information is fully and immediately revealed to buyers in every equilibrium for large enough n , no matter what is the structure of the information. The intensity of price competition implies in fact that the benefits, in terms of expanded market share, of posting a low price - in state L - are too high, when the number of sellers is large enough, even though this means to dissipate any current and future informational rent. This proves to be the case not only with many informed sellers ("collusion" among them to hide their information is very fragile and will not survive when n is large enough), but also when there is just one informed seller, who has then exclusive information.⁵

⁴It follows from our analysis that the characterization of equilibria for information structures between these two is very similar.

⁵In this case, when the prior belief of the uninformed sellers about the state being high is sufficiently low, equilibria (for n large) are characterized by the fact that information is fully

The situation is rather different in the model with clienteles. When all sellers are informed there is again an equilibrium with full and immediate information revelation; however, a collusive equilibrium where (at least part of the) information is not revealed at any date also exists, whatever the number of sellers. Moreover, when only one seller is informed, for most parameter values there are no revealing equilibria, all the equilibria are such that at least part of the information is never revealed, even for n large.

The lack of information revelation in the model with clienteles resembles the one created by the “local monopoly” of the pairwise meetings technology.⁶ Our results show that even allowing for the presence of market signals observed by all traders and the non-exclusivity of information, full revelation of information is not guaranteed. More generally, they suggest that the intensity of competition among sellers proves more effective than the non exclusivity of information in inducing full and immediate disclosure of the agents’ private information, thus shedding some light on the negative message of the pairwise meetings models.

1.1. Related Literature

With complete information, perfect competition arises as a fairly robust limit of game theoretic models (describing both strategic and cooperative interaction among traders) when the number of traders increases.⁷

Even with asymmetric information, if this is of the private values type, the competitive limit result seems to be the norm (see, e.g., Rustichini, Satterthwaite and Williams (1994) in a double auction context). This is true also in decentralized dynamic market structures, where agents’ market power is further constrained by competition over time (Gale (1987), Serrano (2000)). In a dynamic set-up market power may vanish even in the case of monopoly: the Coase conjecture for a durable goods monopolist, uninformed about the demand curve he faces, shows in fact that monopoly pricing, under reasonable assumptions, converges to the competitive price as the frequency of trades increases (see, for example, Gul, Sonnenschein and Wilson (1986)).

revealed to buyers, but not to sellers. All the uninformed sellers in fact immediately post a low price, while the informed still follows a revealing strategy; thus all trade in the high state takes place at the low price.

⁶Note that the trading rules of the pairwise meetings model can be viewed as the limit of those of the model with clienteles as $n \rightarrow \infty$.

⁷See Mas-Colell, Whinston and Green (1995, Chapter 18).

In contrast, when asymmetric information is over sources of aggregate uncertainty, affecting the allocation rule of all agents, as with common values uncertainty, convergence may be problematic. We already mentioned the non-convergence results obtained for models with pairwise meetings. Moreover, Serano, Vohra and Volij (2001) have shown that, even in the presence of private information over sunspot uncertainty, core convergence, a cornerstone of the competitive limit in economies with complete information, may be problematic.

The issue was then also addressed within centralized models of dynamic trading by various papers extending the analysis of Kyle (1985) to allow for more than a single informed trader. As we already noted, the presence of noise traders implies that the relevant issue in such framework is the speed of convergence to the full information prices. Vives (1995) examined a situation with a large number of risk averse informed traders acting myopically, finding that convergence is rather fast (see however Medrano and Vives (2001), where the addition of a large strategic agent to the setup of Vives (1995) limits information revelation). The case of finitely many traders acting strategically on the basis of their private information was analyzed by Kyle (1989) in a static model: he showed the existence of equilibria where only part of the information is revealed and studied how the amount of information revelation varies with the share of informed traders relative to both uninformed and noise traders. Several informed traders were introduced in the dynamic trading model of Kyle (1985) by Holden and Subrahmanyam (1992); they showed, via numerical simulations of the model, that strategic agents compete aggressively by sending very large trade orders, thereby neutralizing the effect of noise traders and leading to quick revelation (in line with our findings for the case in which all sellers are informed). Such results however no longer hold if the signals received by informed traders are only imperfectly correlated, as shown by Foster and Viswanathan (1996) (also via numerical simulations); again there are some analogies with our findings for the case of only one informed seller, in the model without clienteles.

A precise understanding of the conditions under which information revelation and convergence to competitive equilibria occur in dynamic trading models with private information over common values sources of uncertainty is thus far from settled. We believe that our analysis, by clearly identifying such conditions in the set-up of a simple trading model, allows to gain some better understanding of these issues.

Finally, even though formally ours is not a repeated game, we should also mention the relationship between our work (particularly in the case of a single

informed seller) and the literature on two-player repeated games with incomplete information, where partial revelation results in which information is revealed in the first stages are also obtained. See Forges (1992), Zamir (1992), Mertens, Sorin and Zamir (1994) and references therein.

2. The Model

The economy. Agents are of two types: buyers and sellers. There are two commodities, an indivisible consumption good, initially owned by the sellers, and a perfectly divisible commodity ('money'), initially owned by the buyers. There is a continuum of buyers, whose measure is normalized to 1. All buyers are identical: each of them is willing to buy at most one unit of the consumption good. There are n sellers, $1 \leq n < \infty$, and each of them can sell an arbitrarily large number of units of the consumption good. Except possibly for their information, all sellers are also identical.

There is uncertainty over the quality of the indivisible good. Quality can be of two types, high (H) or low (L), and all units of the commodity are of the same quality. Uncertainty can thus be described by two (aggregate) states of the world, H and L . The buyers' valuation for (one unit of) the commodity in the two states is, respectively, u_H and u_L . Similarly, the sellers' valuation (for each unit of the good) is c_H and c_L . Note that the valuation of the commodity is perfectly correlated across agents; we are in a situation of common values uncertainty. We assume that:

$$u_H > c_H > u_L > c_L \geq 0.$$

Thus in each state there are positive gains from trade.

Let us denote by $\alpha_0 \in (0, 1)$ the prior belief, common to all agents in the economy, that the state of the world is H . In what follows, we will always assume that buyers are uninformed over the realization of the state of the world; hence their belief, when trading begins, is given by α_0 . On the other hand, we will allow for different cases with regard to the information sellers have over the realization of the uncertainty.

Trading rules. Trades are organized as follows. There are infinitely many trading dates. At each t , $t = 1, \dots$, every seller simultaneously posts a price at which he is willing to sell the consumption good (to any number of buyers). After observing all the quoted prices, each buyer chooses whether or not to trade in that period. If a buyer accepts an offer made in the period, he buys one unit at

the proposed price and then exits the market. If the buyer rejects, he remains in the market and can then trade at some future date. Sellers remain in the market until all buyers are served, which may never happen. There are no new entrants in the market after the first period. The quality of the good becomes known to buyers only at the end of all trading periods.

When there is more than one seller, $n > 1$, we shall distinguish two possible forms of competition among sellers:

1. *Model without clienteles.* Each period t , following the announcement of the prices posted by the n sellers, every buyer is free to trade with any of them. In this situation sellers compete in prices, under no capacity constraints; competition among sellers is then quite intense (as in the classical model of Bertrand competition).
2. *Model with clienteles.* Each period t , a fraction of size $1/n$ of the buyers who are still in the market is randomly assigned to every seller. In that period each buyer can only buy from the seller he is assigned to. Since buyers observe all n posted prices, and always have the option to refuse to trade, the (temporary) segmentation of the market introduced here does not eliminate price competition among sellers. It only mitigates its intensity, since undercutting prices has a less dramatic effect on the demand for each seller (as in models of Cournot competition, or of Bertrand competition where firms face capacity constraints).⁸ Also, information leakages from an informed seller to all uninformed traders are still possible.

The main features of the economy we described are then a market where all the units sold are of the same quality, buyers and sellers are separated and clearly identified, trade may be repeated over time, and there are reasons for trade other than informational differences. We can say that such features characterize situations like that of a specific wine crop sold in a fair by the producers, or the market of derivatives on (aggregate) credit risk.⁹

⁸It should be clear from the analysis which follows that the details of the specification of the model with clienteles play no essential role. The substance of our results extends in fact to various other trading rules, as long as they imply that any seller is unable to instaneously reach all buyers.

⁹Along the same line, we can think of clienteles as buyers being inside the kiosk of a wine seller. In a given moment, they can buy only from that seller, although information flows across kiosks. No clienteles would correspond to all buyers being located outside of every kiosk.

Both buyers and sellers evaluate payoffs from future trades according to the common discount factor $\delta \in (0, 1)$. In this paper we are more interested in the limiting results as $n \rightarrow \infty$ rather than as $\delta \rightarrow 1$. However, all our results hold for any $\delta < 1$ sufficiently large. We should think in fact of trading dates as taking place with a very high frequency; the reader is then invited to fix δ at some arbitrarily large value.

In most of the paper we focus our attention on the case where sellers can propose only one of two given prices, a high price $p_H \in [c_H, u_H)$ and a low price $p_L \in (c_L, u_L]$.¹⁰ This parsimonious formulation allows us to provide a complete characterization of all the equilibria of the trading game, and to better focus on the issues concerning information revelation we are primarily concerned with. Note that, if the state was observable by all traders (with full and symmetric information), all possible trades would take place at the initial date, at the price p_H in state H and p_L in state L . More importantly, we will show that most of the qualitative properties we derive for the equilibria remain valid also when sellers are free to propose any price in \mathbb{R}_+ .

Various cases with regard to the number of sellers in the economy and their information will be considered. This will allow us to disentangle in the revelation of information the role of market power given by the sellers' size from that given by their private information. In each case we characterize the perfect Bayesian equilibria of the trading game described above. From now on we refer to a perfect Bayesian equilibrium simply as an equilibrium.

As a benchmark case, we begin by analyzing the model with an informed monopolist in Section 3. In Section 4 we examine the model without clienteles and in Section 5 the model with clienteles. In both cases, we analyze the properties of the equilibria as the number of sellers gets large and perform our study under the following two extreme information structures. First, one where all sellers are informed. In this case, as n grows sellers are both “informationally small” and their market power as determined by their size becomes negligible. Second, we analyze the polar opposite case: there are various sellers, but only one of them is informed. Now, as n gets large, this seller remains “informationally large”, while his size relative to the market decreases (as that of all sellers), so that information is the only possible source of market power. The welfare properties of the equilibria are then discussed and compared in Section 6.

¹⁰Allowing for the knife-edge cases $p_H = u_H$ or $p_L = c_L$ would alter some of the following results (though in a rather inessential way).

3. Monopoly

Consider the case where there is only one seller ($n = 1$), who is fully informed of the realization of the uncertainty.

The following observations will allow us to simplify the definition of an equilibrium. Note first that when the seller observes state H (i.e., the seller is of 'type H ') he will always propose p_H at any trading date. Also, whenever a buyer is proposed a price p_L he will always accept, no matter what his belief is over the realization of the uncertainty.

On the other hand, the 'type L ' seller faces a non-trivial choice between offering p_H and p_L ; similarly the buyers, when they are proposed p_H , have to decide whether to accept or reject. In both cases we will allow for the possibility that the agents may randomize in their choice. Let then $q_S(t)$ denote the probability that the seller in state L proposes p_H at date t (given histories according to which in all previous periods the seller always proposed p_H and at least some buyers always rejected; similarly, let $q_B(t)$ be the probability that a typical buyer accepts, at date t , if the seller proposes p_H , given histories in which the seller always proposed p_H in the past.¹¹ The payoff of a buyer in a period where trade takes place, say at price p_H in state L , is $u_L - p_H$; for the seller it is then $p_H - c_L$ times the fraction of buyers accepting.

Each period t , after observing the price proposed by the seller, buyers will also update their belief over the state of the world. If the seller proposes p_L the buyers inference is irrelevant since, as already argued above, their optimal action is always to accept. Let α_t denote then the buyers' belief at date t that the state of the world is H if the seller proposed p_H at t and in all past periods; such belief is updated every period, using Bayes' rule and taking into account the strategy of the type L seller:

$$\alpha_t = \frac{\alpha_{t-1}}{\alpha_{t-1} + (1 - \alpha_{t-1})q_S(t)} \text{ for all } t \geq 1 \quad (3.1)$$

Note that α_t is always weakly increasing with t , and is strictly increasing as long as $q_S(t) < 1$.

An *equilibrium* of the trading game with an informed monopolist is then described by the sequences $\{q_B(t)\}_{t \geq 1}$, $\{q_S(t)\}_{t \geq 1}$, $\{\alpha_t\}_{t \geq 1}$ such that:

¹¹Additional equilibria can be found in which all buyers play pure strategies, and a proportion $q_B(t)$ accepts the high price at date t , while the rest reject it. These equilibria are outcome equivalent to the ones we study. This is the sense in which there is no loss of generality in restricting attention to symmetric equilibria, as we shall do.

- (i) $\{\alpha_t\}_{t \geq 1}$ satisfies (3.1);
- (ii) at each t , after every partial history in which not all buyers accepted the seller's offer in one of the previous periods, $\{q_B(\tau)\}_{\tau \geq t}$ maximizes the buyers' discounted (to that date) payoff, given $\{q_S(\tau), \alpha_\tau\}_{\tau \geq t}$, and $\{q_S(\tau)\}_{\tau \geq t}$ maximizes the type L seller's discounted payoff, given $\{q_B(\tau)\}_{\tau \geq t}$.

We will provide a complete characterization of the equilibria of this game. It will be shown that we never have complete revelation of the seller's information. In particular, if the prior belief α_0 that the true state of the world is H is sufficiently high, no information is ever revealed in the trading process: all equilibria exhibit perfect pooling (of the two seller's types) and no delay (all trades take place at the initial date). On the other hand, if the buyers' prior belief over H is not high enough, some information gets always revealed in the first trading date (as, with some positive probability, the type L seller will propose a revealing price, p_L). But it will be the minimum amount of information needed to induce buyers to be willing to trade at a high price. After that is accomplished, no further information is ever revealed. In this situation we have 'partial pooling'. In addition there is delay: while p_L is always immediately accepted by buyers, when p_H is proposed the buyers will reject, in all periods, with some positive probability. Delay can even be infinite.

Define $\bar{\alpha}$ to be such that $\bar{\alpha}u_H + (1 - \bar{\alpha})u_L = p_H$. That is, $\bar{\alpha}$ is the belief that makes buyers exactly indifferent between trading at p_H with probability 1 and not trading at all. Formally, we have the following:

Proposition 1. *In the model with an informed monopolist the following equilibria obtain:*

- (i) **No information revelation:** when $\alpha_0 \geq \bar{\alpha}$, in any equilibrium, we have $q_S(t) = 1$ for all t . In particular, if $\alpha_0 > \bar{\alpha}$, the equilibrium is unique with $q_B(t) = 1$ for every t . On the other hand, if $\alpha_0 = \bar{\alpha}$, there are also equilibria where $q_B(t) \in (0, 1]$ for all $t \geq 1$.
- (ii) **Partial and immediate revelation:** when $\alpha_0 < \bar{\alpha}$, in all equilibria, we have $q_S(1) \in (0, 1)$ so that $\alpha_1 = \bar{\alpha}$, $q_S(t) = 1$ for all $t > 1$, $q_B(t) \in (0, 1]$ for all $t \geq 1$.

To gain some intuition on the result, notice that in our set-up the profits a monopolist seller can get from his private information and his market power lie in the possibility of manipulating buyers' beliefs and induce buyers to agree to

trade at a high price, with minimal delay. Our result shows that the seller will always succeed in generating such beliefs, by revealing the minimal amount of information (possibly zero) which is necessary, and all in the first period. There is however a cost, given by the fact that there may be (possibly considerable) delay in trade; the need for such cost comes from the fact that the seller is otherwise unable to credibly commit to reveal his information.

To prove the proposition we will establish first some preliminary results.

A) Full and immediate information revelation never occurs (in finite time); at no date the seller in state L proposes p_L with probability 1 :

Lemma 1. *At any equilibrium, $q_S(t) > 0$ for every t .*

Proof of Lemma 1: Suppose not: there exists a period t such that $q_S(t) = 0$. Since, if the state is H , the seller proposes p_H for certain, it follows that there is full separation at t . Thus we have $\alpha_t = 1$. Upon observing p_H at t the buyers can infer the state is H for certain and so their optimal strategy must always be to accept p_H with probability 1 at date t ($q_B(t) = 1$). But then the optimal strategy of the seller in state L at t would be to propose p_H rather than p_L , a contradiction. ■

B) If, at some t , the buyers' strategy is to accept p_H for sure, then in that period the type L seller will propose p_H for certain. This in turn implies, when the buyers' belief α_t is sufficiently close to 1, that at each earlier date the buyers should also prefer to accept p_H for sure; hence the seller must prefer to propose p_H for certain. Hence we have:

Lemma 2. *If, at an equilibrium, $q_B(t) = 1$ for some t , then we must also have $q_S(t) = 1$. If, in addition, $\alpha_t > \bar{\alpha}$ we obtain that, for every $t' < t$, $q_B(t') = q_S(t') = 1$.*

Proof of Lemma 2: This is easily established by backwards induction. If in period t , $q_B(t) = 1$, the seller's best response in that period is clearly $q_S(t) = 1$. Since $q_S(t) = 1$, $\alpha_{t-1} = \alpha_t$; thus, if $\alpha_t > \bar{\alpha}$ the buyers' payoff from accepting at $t - 1$ is positive and strictly higher than the payoff from accepting at t , for all $\delta < 1$. The buyers' best response in period $t - 1$ must then be $q_B(t - 1) = 1$. Iterating the argument, we find that the same must be true at all previous dates $t' < t$. ■

C) If in equilibrium buyers randomize for infinitely many periods, their belief α_t must jump to a sufficiently high level at the initial date and stay constant at that level forever after:

Lemma 3. *If, at every date t , $q_B(t) \in (0, 1)$, then we must have $\alpha_t = \bar{\alpha}$ for all t .*

Proof of Lemma 3: Note that the evolution of posterior beliefs α_t is determined from Bayes' rule using the seller's strategy and, accordingly, follows equation (3.1). As we already noticed, this sequence of posterior beliefs is non-decreasing in t . In addition, to sustain the buyers' randomization in every period, we need the following condition to hold at all t :¹²

$$\alpha_t u_H + (1 - \alpha_t) u_L - p_H = \delta[(1 - \alpha_t)(1 - q_S(t + 1))(u_L - p_L) + (1 - (1 - \alpha_t)(1 - q_S(t + 1)))(\alpha_{t+1} u_H + (1 - \alpha_{t+1}) u_L - p_H)] \quad (3.2)$$

where on the left hand side we have the payoff from accepting p_H and, on the right hand side, the payoff from rejecting it at t and accepting it at $t + 1$.

Because by hypothesis, the randomization involves infinitely many periods, the infinite sequence of posteriors $\{\alpha_t\}$, which is monotone and bounded, has a limit. We will show that $\lim_{t \rightarrow \infty} \alpha_t = \bar{\alpha}$.

Suppose $1 > \lim_{t \rightarrow \infty} \alpha_t > \bar{\alpha}$; in this case, by equation (3.1), $\lim_{t \rightarrow \infty} q_S(t) = 1$. But if we consider (3.2) and take the limit as $t \rightarrow \infty$, plugging $\lim_{t \rightarrow \infty} q_S(t) = 1$ in and using the fact that $\lim_{t \rightarrow \infty} (\alpha_t - \alpha_{t+1}) = 0$, we reach a contradiction, because for $\delta < 1$ the term on the right hand side of (3.2) will be strictly lower than the one on the left hand side, so buyers strictly prefer to accept.

Similarly, if $\lim_{t \rightarrow \infty} \alpha_t = 1$: the term on the left of (3.2) tends to $u_H - p_H$ while the term on the right tends to $\delta(u_H - p_H)$, which is strictly lower, thus again we get a contradiction.

It remains then to show that we cannot have $\lim_{t \rightarrow \infty} \alpha_t < \bar{\alpha}$ either. But this is immediate, since the expected payoff from accepting a p_H offer, $p_H - \alpha_t u_H + (1 - \alpha_t) u_L$, is negative whenever $\alpha_t < \bar{\alpha}$. Thus buyers would be better off by refusing to trade altogether at t , instead of randomizing between accepting and rejecting.

Finally, note that, in order to sustain the randomization of the buyers at any date t , it must be $\alpha_t \geq \bar{\alpha}$. This fact, together with the property $\lim_{t \rightarrow \infty} \alpha_t = \bar{\alpha}$ established above, implies that infinite randomization of the buyers requires $\alpha_t = \bar{\alpha}$ for all t . ■

¹²The same arguments apply if the randomization does not involve two consecutive periods. The right hand side of (3.2) is then more involved, but the essence of the argument is identical.

We are now ready to prove the main result of the section:

Proof of Proposition 1: By Lemma 1, pure strategies where $q_S(t) = 0$ for some t can never be part of an equilibrium. Consider then the only other possible pure strategy of the type L seller, $q_S(t) = 1$ for all t .

If $\alpha_0 \geq \bar{\alpha}$, a best reply for the buyers to this strategy is $q_B(t) = 1$ for all t : their expected payoff is in fact non-negative and any other strategy would only induce delay and still result in either no trade or trade at the same price, p_H , yielding so a lower (weakly if $\alpha_0 = \bar{\alpha}$) payoff. The strategy $q_S(t) = 1$ for all t is then also the seller's best reply to $q_B(t) = 1$ for all t since the payoff obtained by the type L seller is $p_H - c_L$, the highest possible. This establishes that $q_B(t) = q_S(t) = 1$ for all t is an equilibrium if $\alpha_0 \geq \bar{\alpha}$, as claimed in part (i) of the statement of the proposition.

On the other hand, if $\alpha_0 < \bar{\alpha}$, the buyers' best reply to $q_S(t) = 1$ for all t is $q_B(t) = 0$ for all t ; but then the seller's best reply is $q_S(t) = 0$ for all t , so that we do not have a pure strategy equilibrium in this case.

Consider next the candidate equilibria where buyers randomize for infinitely many periods. From Lemma 3, such equilibria require the seller to - possibly - randomize at the initial date, so as to induce the posterior belief $\alpha_1 = \bar{\alpha}$, and to propose p_H with probability 1 at all later dates $t > 1$. This is clearly possible only if $\alpha_0 \leq \bar{\alpha}$ (hence, when $\alpha_0 > \bar{\alpha}$ these equilibria never exist).

Let us denote by $V_L(t)$ the present value, at t , of the discounted expected flow of payoffs of the type L seller, given that he always proposed p_H in the past (including the current period t), and got always rejected; $V_L(t)$ satisfies then the following:

$$V_L(t) = q_B(t)(p_H - c_L) + \delta(1 - q_B(t))V_L(t + 1) \quad (3.3)$$

Any sequence of values $q_B(t) \in (0, 1)$ satisfying the conditions:

$$\begin{aligned} p_L - c_L &= q_B(1)(p_H - c_L) + \delta(1 - q_B(1))V_L(2) \\ p_L - c_L &\leq q_B(t)(p_H - c_L) + \delta(1 - q_B(t))V_L(t + 1) \text{ for all } t > 1 \end{aligned} \quad (3.4)$$

supports the strategy $q_S(1) \in (0, 1)$, $q_S(t) = 1$ for all $t > 1$ as the seller's best response to $\{q_B(t)\}_t$. It is immediate to verify that we can always find some, in fact many, sequences with this property. Furthermore, since $\alpha_t = \bar{\alpha}$ for all $t \geq 1$, any sequence of values $q_B(t) \in (0, 1)$ is a best reply for the buyers. Thus, as stated in part (ii) of the statement of the proposition, such equilibria always exist if $\alpha_0 < \bar{\alpha}$. By a similar argument, any sequence of values $q_B(t) \in (0, 1)$ satisfying

the inequality in the second equation of (3.4) for all $t \geq 1$ and the seller's strategy $q_S(t) = 1$ for all $t \geq 1$ are a best response to each other (and hence constitute an equilibrium) when $\alpha_0 = \bar{\alpha}$.

To complete the proof of the Proposition, it remains to consider the possibility of an equilibrium where buyers randomize for a positive but finite number of periods.¹³ Suppose that there is an equilibrium where $q_B(t) = 0$ for all t greater or equal than some date $T \geq 2$. Then the seller's best response, as we already argued, would be $q_S(t) = 0$ for all $t \geq T$, which by Lemma 1 cannot be part of an equilibrium.

On the other hand, if $q_B(t) = 1$ for all $t \geq T \geq 2$ is part of an equilibrium strategy, we must have $\alpha_T \geq \bar{\alpha}$. If $\alpha_T > \bar{\alpha}$ we reach again a contradiction, by Lemma 2. If $\alpha_T = \bar{\alpha}$ and $\alpha_t < \bar{\alpha}$ for $t < T$, we must have $q_S(T) \in (0, 1)$; for this choice of the seller to be optimal we need, from (3.4), taking into account that $q_B(T) = 1$, the following equality to hold, $p_L - c_L = p_H - c_L$, which is impossible. We are then left with the case where $\alpha_T = \bar{\alpha}$ and, for some $\bar{t} < T$, $\alpha_t = \bar{\alpha}$ for $\bar{t} < t < T$, so that $q_S(t) = 1$ for all $t > \bar{t}$, $q_S(\bar{t}) \in (0, 1)$. Note first that this can only be part of an equilibrium if $\bar{t} = 1$ and hence $\alpha_t = \bar{\alpha}$ for all $t \geq 1$.¹⁴ The conditions for the optimality of the seller's strategy are again given by (3.4), where $V_L(2)$ is defined recursively by (3.3) together with the equality $V_L(T - 1) = p_H - c_L$. It is easy to check that, for any $\delta < 1$ we can find T , sufficiently high, such that these conditions are satisfied for some sequence $\{q_B(t)\}_{t \geq 1}$ exhibiting the property $q_B(t) = 1$ for all $t \geq T$. The closer is δ to 1, the larger is the minimal number of periods of randomization T required.

We conclude that, when $\alpha_0 \leq \bar{\alpha}$, there exist equilibria where buyers randomize both for an infinite and a finite number T of periods, with T larger the closer is δ to 1. On the other hand, if $\alpha_0 > \bar{\alpha}$, there are no equilibria where buyers randomize. ■

3.1. Welfare

We proceed next to analyze the welfare properties of the equilibria we found. We will show that, in our set-up, (interim) inefficiencies are identified with the presence of delay in trading. This is true even if we take into account that information

¹³It is immediate to see that we can only have an equilibrium where buyers never randomize if the seller also never randomizes, the case already considered at the beginning of the proof.

¹⁴If $\bar{t} > 1$, so that $\alpha_t < \bar{\alpha}$ and hence $q_B(t) = 0$ for $t < \bar{t}$, from the inequality in the second equation of (3.4) we obtain $p_L - c_L \leq \delta V_L(t+1)$ for $t < \bar{t}$. But this is impossible since $q_S(\bar{t}) \in (0, 1)$ implies that $V_L(\bar{t}) = p_L - c_L$.

is private, and require (interim) improvements to satisfy incentive compatibility restrictions.

- (i) The equilibria with perfect pooling and immediate acceptance by the buyers, obtained when $\alpha_0 \geq \bar{\alpha}$, are clearly interim Pareto efficient: all gains from trade are exhausted, with no delay.¹⁵

The equilibrium allocation yields in fact the highest possible payoff the seller can attain in the model; any interim improvement for the seller would have to be based on trades at a price higher than p_H , which would make buyers worse off. Moreover, at this equilibrium, both types of the seller and the buyers attain a strictly positive payoff. Let U_H, U_L, U_B be the present value, at $t = 1$, of the discounted expected flow of payoffs of the two types of seller and the buyers; we have $U_H = p_H - c_H$, $U_L = p_H - c_L$, $U_B = \alpha_0 u_H + (1 - \alpha_0)u_L - p_H$.

- (ii) On the other hand, the equilibria with delay (obtained when $\alpha_0 \leq \bar{\alpha}$) are always interim incentive inefficient.

Since delay is costly, they are obviously interim inefficient. But we will show that we can always construct an incentive compatible mechanism that interim dominates the equilibrium. This is true even though - when $\alpha_0 < \bar{\alpha}$ - some information is revealed in the first trading date (there is partial pooling). To see this, consider the following. Each period t the seller always charges p_H in state H and p_L in state L ; all buyers immediately accept when p_L is offered and reject with probability $q_B(t)$ at each period t when p_H is proposed, where $\{q_B(t)\}_{t \geq 1}$ is the same as in the equilibrium. The specification of the buyers' behavior ensures the incentive compatibility of the prescribed choice of the seller and hence of the mechanism proposed; we immediately see that such mechanism leaves the two types of monopolist indifferent, but strictly improves the buyers¹⁶.

From the equality condition in (3.4), needed to sustain the type L seller's randomization in $t = 1$, it follows that his payoff at this other equilibrium is $U_L = p_L - c_L$, which must equal the payoff obtained by choosing p_H at $t = 1$. The latter is obtained by iterating the term on the right hand side

¹⁵As with any other equilibrium, the allocation obtained is clearly incentive compatible (this is even clearer in this case, since the allocation is constant); hence, a fortiori, it is also interim incentive efficient.

¹⁶At $t = 1$ in fact buyers no longer make any trade at a 'wrong' price (p_H in state L), not ex post individually rational.

in (3.4), from which we obtain that the payoff of the seller in state L must also be:

$$U_L = \sum_{\tau=1}^{\infty} [\delta^{\tau-1} q_B(\tau) (\prod_{l=1}^{\tau-1} (1 - q_B(l)))] (p_H - c_L).$$

The type H seller's payoff is then readily obtained:

$$U_H = \sum_{\tau=1}^{\infty} [\delta^{\tau-1} q_B(\tau) (\prod_{l=1}^{\tau-1} (1 - q_B(l)))] (p_H - c_H).$$

Hence $U_H = U_L \left(\frac{p_H - c_H}{p_H - c_L} \right) < U_L$. Since the probability of being able to trade at a price p_H is the same for the H and the L type of seller and the valuation of type H is higher, it follows that his payoff at equilibrium is strictly lower than the payoff of the type L seller.¹⁷ Finally, the buyers' payoff is

$$\begin{aligned} U_B = & \alpha_0 \sum_{\tau=1}^{\infty} [\delta^{\tau-1} q_B(\tau) (\prod_{l=1}^{\tau-1} (1 - q_B(l)))] (u_H - p_H) \\ & + (1 - \alpha_0) q_S(1) \sum_{\tau=1}^{\infty} [\delta^{\tau-1} q_B(\tau) (\prod_{l=1}^{\tau-1} (1 - q_B(l)))] (u_L - p_H) + \\ & (1 - \alpha_0)(1 - q_S(1))(u_L - p_L) = (1 - \alpha_0)(1 - q_S(1))(u_L - p_L) \end{aligned}$$

where the last equality follows from the specification of $q_S(1)$ at the equilibrium considered.

Although the comparison among the equilibrium payoffs of buyers and sellers in the equilibria of type (i) and (ii) (without and with delay) involves agents in different economies, characterized by different values of the prior belief α_0 , it is still interesting to note that both types of the seller have a strictly lower payoff in the second equilibrium than in the first. Thus, information revelation comes at a cost to the seller.

Our findings here, that interim (incentive) inefficiencies are associated with lack of information revelation, while full and immediate information revelation leads to efficiency, are analogous to those of the literature on pairwise meetings (see Blouin and Serrano (2001)) and others. However, as we will see more clearly in the what follows (in particular Lemma 2 and Section 6.1.1), the relationship between efficiency and information revelation is more subtle in the present framework.

¹⁷Note that the same is also true in the perfect pooling equilibria considered in (i).

3.1.1. Asymptotic inefficiency

As we stated earlier, all our results remain valid for any large enough value of δ . In particular, inefficiencies may persist even as $\delta \rightarrow 1$. To illustrate this claim, we will show:

Lemma 4. *Let $\delta \rightarrow 1$; there exist equilibrium sequences such that all equilibria are interim incentive inefficient and the efficiency loss persists in the limit.*

Proof of Lemma 4. Consider the equilibrium with partial revelation. Notice that the available interim surplus is

$$W = \alpha_0(u_H - c_H) + (1 - \alpha_0)(u_L - c_L).$$

Recall that the payoff of the seller in state L equals:

$$U_L = p_L - c_L,$$

the one in state H is

$$U_H = (p_H - c_H) \frac{p_L - c_L}{p_H - c_L},$$

and both are independent of δ . The buyers' expected payoff is then

$$U_B = (1 - \alpha_0)(1 - q_S(1))(u_L - p_L)$$

which, using the fact that $\alpha_1 = \bar{\alpha}$ implies $q_S(1) = \frac{\alpha_0(1-\alpha_1)}{\alpha_1(1-\alpha_0)}$, simplifies as follows, $U_B = (\bar{\alpha} - \alpha_0)(u_L - p_L)/\bar{\alpha}$, also independent of δ . It is then straightforward to check that $\alpha_0 U_H + (1 - \alpha_0)U_L + U_B < W$. Thus there is a positive - and constant - welfare loss for all $\delta < 1$. ■

The persistence of inefficiency even when $\delta \rightarrow 1$ (so that the cost of delaying trade by any finite number of periods become vanishingly small) has to be imputed to the fact that, as δ approaches 1, a significant fraction of trades takes place with longer and longer (possibly infinite) delay. To see this, consider the case where buyers randomize every period according to a constant probability, i.e., $q_B(t) = q_B$ for all t ; q_B is then pinned down by the condition needed to sustain the randomization of the seller in state L : $p_L - c_L = (p_H - c_L) \frac{q_B}{1 - \delta(1 - q_B)}$, yielding

$$q_B = \frac{1 - \delta}{1 - \delta \frac{p_L - c_L}{p_H - c_L}}$$

which clearly tends to 0 as $\delta \rightarrow 1$.

3.2. Robustness

The results obtained on the characterization of the equilibria are robust to the extension of the model along several dimensions.

3.2.1. Continuum of prices

The characterization of the equilibria was obtained under the simplifying assumption that sellers can only propose one of two possible prices, p_H and p_L , exogenously given. We show here that the main qualitative properties of equilibria, in particular with regard to their information and efficiency properties, remain essentially the same when sellers are free to propose any price in a closed interval, subset of (c_L, u_H) . Thus, for the issues we have studied, our simplifying assumption is without essential loss of generality.

- a) We still have no equilibrium sequence with separation in finite time, i.e., the seller charging different prices at some t in H and L with probability 1.¹⁸

The result follows by a straightforward extension of Lemma 1. Suppose such an equilibrium existed. In state H the seller would offer a price $\bar{p} \geq c_H$ (sales at any price below c_H would in fact result in losses). If buyers accept such price for sure, the seller in state L would prefer to also charge \bar{p} , rather than the separating price, which is some price \underline{p} less or equal than u_L , a contradiction. But there are no equilibria either where the high price is always rejected with some positive probability: such randomization on the part of buyers requires in fact, after period t (when information is revealed), a decreasing sequence of prices in state H , which at some point would have to fall below c_H , again a contradiction.¹⁹

- b) If the prior belief α_0 is high enough, so that we can find prices (typically an interval $I \subseteq [c_H, u_H)$) at which both types of the seller as well as buyers are willing to trade, pooling equilibria with immediate trading again exist, though now there is a continuum of them.

For any $p \in I$, there is in fact an equilibrium where the seller charges p both in state H and L and buyers accept; if the seller deviates to $p' \neq$

¹⁸This stands in contrast with the result in Laffont and Maskin (1990) who show that a separating equilibrium always exists, in the set-up of a one period trading model where the uninformed's demand is strictly decreasing in the price.

¹⁹Note that if u_H were allowed as a possible price an equilibrium of this type would exist: set $\bar{p} = u_H$, $\underline{p} = u_L$, and $q_B(t)$ constant and low enough to deter deviations of the state L seller.

p , off-equilibrium beliefs are such that $\alpha_1 = 0$, thus buyers accept p' if and only if $p' \leq u_L$, which ensures that no profitable deviation exists for any type of seller. Having this continuum of equilibria, the complicated issue of equilibrium coordination arises, which we avoid with our simplifying assumption.

- c) If α_0 is small enough so that there is no price at which buyers and the seller in both states are willing to trade, the seller's strategy in state L needs to be such that some information is revealed, as in the case considered before with only two prices.

Therefore, equilibria are always characterized by the fact that the state L seller randomizes in the initial period between two prices. The low price is always given by u_L . (To see this, note that following such a low price announcement we have a complete information game between the state L seller and the buyers. In such a game, u_L is the only equilibrium price.) The high price is some price above c_H , that is also charged by the seller in state H .²⁰ Buyers randomize then between accepting and rejecting.

- c1) We always have equilibria where, at all subsequent dates $t > 1$, the state L seller no longer randomizes and proposes the high price, while buyers keep randomizing between accepting and rejecting.

Such equilibria have then the same features as the equilibrium with partial information revelation we obtained in the previous section for the case of low α_0 . Again there is a continuum of such equilibria, for the different values that the high price can take.

- c2) There may be other equilibria too, where the seller (in state L) randomizes for an arbitrary, possibly infinite, number of periods.

At all equilibria where the state L seller randomizes for more than one period (so that $\alpha_{t+1} > \alpha_t \geq \bar{\alpha}$) buyers have to randomize and, by essentially the same argument as in the proof of Lemma 2, they have to do it at all periods. This will in turn be optimal provided the following condition holds:

$$\alpha_t u_H + (1 - \alpha_t) u_L - p_H(t) = \delta(1 - (1 - \alpha_t)(1 - q_S(t+1))) (\alpha_{t+1} u_H + (1 - \alpha_{t+1}) u_L - p_H(t+1)), \quad (3.5)$$

²⁰It is possible that the seller randomizes in state H too (and the state L seller randomizes over the low price and more than one level of the high price). The argument in the text easily extends to this case.

where $p_H(t)$ denotes the price charged by the seller in state H with probability 1 and in state L with probability $q_S(t)$, which may now vary with t . Since the sequence of posterior beliefs $\{\alpha_t\}_{t \geq 1}$ must converge and the prices in state H , $\{p_H(t)\}_{t \geq 1}$, cannot fall below c_H , as t tends to infinity, condition (3.5) can only hold if, for large t , we have

$$\alpha_t u_H + (1 - \alpha_t) u_L - p_H(t) \cong \alpha_{t+1} u_H + (1 - \alpha_{t+1}) u_L - p_H(t+1) \cong 0. \quad (3.6)$$

Note that $\alpha_t u_H + (1 - \alpha_t) u_L - p_H(t) \geq 0$ for all t and (3.5) implies that the sequence $\{\alpha_t u_H + (1 - \alpha_t) u_L - p_H(t)\}_t$ is increasing; hence from (3.6) we get:

$$\alpha_t u_H + (1 - \alpha_t) u_L - p_H(t) = 0 \text{ for all } t, \quad (3.7)$$

i.e., buyers are always indifferent between trading and not trading at $p_H(t)$, as in the equilibrium with one period randomization we found above in c1). Moreover, from (3.7) it follows that the price $p_H(t)$ proposed in state H increases over time.

Thus we can have information being revealed now over many periods. However, it is important to notice that, as shown below, at all such equilibria the payoffs of buyers and the seller are either equal or lower to the ones obtained in the equilibria where the seller in state L randomizes for only one period. The equilibria with more periods of information revelation (as in c2) are then interim Pareto dominated by the equilibrium with only one period of randomization (as in c1). More information revelation requires greater delays in trading and hence entails a welfare loss.

Lemma 5. *At all equilibria where the state L seller randomizes for more than one period (as in c2), the payoff of buyers and the state L seller is the same while the payoff of the state H seller is lower than in an equilibrium where the state L seller randomizes for one period only (as in c1).*

Proof of Lemma 5. From the above description it is immediate to see that the present value of the discounted expected flow of payoffs at an equilibrium as in c2) is $u_L - c_L$ for the seller in state L and 0 for buyers (since, as we argued, $p_L = u_L$), thus for both it is the same as in the equilibrium in c1). The payoff for the state H seller has then also the same expression as in such equilibrium²¹,

²¹It is immediate to verify that the expression for U_H obtained in section 3.1 can be equivalently written in this form.

$p_L - c_L - \sum_{\tau=1}^{\infty} [\delta^{\tau-1} q_B(\tau) (\prod_{l=1}^{\tau-1} (1 - q_B(l)))] (c_H - c_L)$, though its value will typically be different (since buyers may randomize with different probabilities): in particular, we can show it will be lower. This follows from the fact that, as we can see from (3.4), to sustain the state L seller's randomization with an increasing price sequence $\{p_H(t)\}_{t \geq 1}$ buyers have to reject with higher probabilities ($q_B(\cdot)$ will be lower) and these are the only variables determining the state H seller's payoff, as we see from the previous expression. ■

3.2.2. Finite horizon

All the equilibria we obtained for the case of infinitely many trading dates remain (or are approximated by) equilibria when there is only a finite number of trading dates ($\mathbf{T} \geq 1$). This is clear when the equilibrium in question involves a finite number of periods of trade; but even when there are infinite periods of trade, for example with infinite randomization on the part of buyers, for any \mathbf{T} there is an analogous equilibrium where buyers randomize at each trading date.

To illustrate, consider the one-period ($\mathbf{T} = 1$) version of the model. In this case, rejection of a price leads to “no trade”, thus to a zero payoff. It is easy to see that if $\alpha_0 > \bar{\alpha}$, the unique equilibrium continues to yield trade at p_H in both states. If $\alpha_0 < \bar{\alpha}$, the unique equilibrium prescribes that the seller in state L randomizes between p_H and p_L so as to induce the belief $\alpha_1 = \bar{\alpha}$, while the buyers must accept the high price with probability $q_B(1)$ (where $q_B(1)$ is set at a level such that the seller in L is exactly indifferent between charging p_L and p_H).

When $\mathbf{T} = 2$, the equilibrium with $\alpha_0 > \bar{\alpha}$ continues to have pooling on p_H and immediate trade, with no delay. When $\alpha_0 < \bar{\alpha}$, with $\mathbf{T} = 2$ we continue to have as the unique equilibrium the one in which $q_S(1)$ is chosen to yield $\alpha_1 = \bar{\alpha}$, while $q_B(1)$ and $q_B(2)$ are chosen to make the seller in state L indifferent between charging p_L and p_H at $t = 1$ and to (weakly) prefer p_H at $t = 2$; the information is then partially revealed only in period 1.

We can also show that no other equilibria exist, in particular there is no equilibrium where both buyers and the seller in state L randomize at each trading date: in that case in fact we must have $\alpha_t > \bar{\alpha}$ for all t , thus buyers at the terminal date accept p_H for sure, and so the seller always proposes p_H at \mathbf{T} , but then buyers cannot be indifferent between accepting and rejecting p_H at $\mathbf{T} - 1$.

We thus conclude that the equilibria are essentially the same when $\mathbf{T} < \infty$ and $\mathbf{T} = \infty$, our findings are not generated by a possible discontinuity at $\mathbf{T} = \infty$.

4. Oligopoly without Clienteles

In this section we examine the case where there are $n > 1$ sellers, who compete in prices among them, in the absence of clienteles. As described in Section 2, this means that each period sellers simultaneously announce a price in the set $\{p_H, p_L\}$, buyers get to observe the list of announced prices and the identity of the seller behind each price and can freely choose whom to trade with. Hence, if both prices are called on by firms, those who announce p_L split the entire market equally among them, while those announcing p_H sell no units. In this situation, competition is quite fierce since each seller, by undercutting, can steal immediately its competitors' market share.

We will explore this model, as well as its counterpart with clienteles in the next section, under two extreme information structures: one in which all sellers are informed regarding the true state of the world and the other where only one of them is informed.

4.1. All Informed Oligopolists

As in the case of monopoly, state H sellers will always charge p_H and, as soon as the price p_L is in the list of announced prices, all buyers remaining in the market will always buy at this price. If, on the other hand, all sellers announced p_H in the first t periods, the buyers' choice at t depends on their belief α_t that the state of the world is H . Let $q_B(t)$ denote then the probability that a buyer accepts, at t , if all sellers propose p_H , given histories where all sellers offered p_H in the past. The formal definition of the strategies of buyers and sellers - and hence of an equilibrium - is otherwise the same as in the previous section (we denote by $q_S(t)$ the probability that each state L seller charges p_H in period t); similarly, α_t is the buyers' belief at t that the state is H if all sellers proposed p_H at t and in each prior period.²²

The non-exclusivity of the sellers' information, as well as the limits on their market power given by the presence of various sellers competing among them, impose severe constraints on the sellers' ability to hide their information and manipulate buyers' beliefs as in the case of monopoly. To hide the information would in fact require to repeatedly announce a high price even when the state

²²Again we can restrict our attention, without loss of generality, to symmetric equilibria. The purification argument outlined above (in footnote 11) continues to apply to the buyers, and by the structure of the model, it is easy to see that there are no asymmetric equilibria among sellers. Hence the notation $q_S(t)$ just presented.

is L . However, doing so now would give any other seller a strong incentive to undercut. Even though by undercutting the seller would reveal its information, the benefits from expanding its market share would be higher the larger is the number n of sellers; hence, for n sufficiently large they will outweigh the costs of revealing the information.

We proceed to characterize the equilibria of the model in this case. We will show that, due to the non-exclusivity of the information that each seller holds, there is an equilibrium, where all the information is immediately revealed: each seller in period 1 announces p_H in state H and p_L in state L with probability 1, while buyers always accept both p_L and p_H (if all sellers offered p_H). When the number of sellers n is small enough, we can also have 'collusive' equilibria, where sellers behave as in the equilibrium with monopoly obtained in the previous section, or where they randomize in state L for the first T periods and then always propose p_L . However, as n grows, all these 'collusive' strategies cease to be part of equilibria, and there is no other equilibrium than full and immediate information revelation. Formally,

Proposition 2. *In the model with n informed sellers without clienteles:*

(i) **Full and immediate information revelation:** for any $n \geq 2$, there always exists an equilibrium where $q_S(1) = 0$ and $q_B(1) = 1$.

(ii) For n small enough, the following equilibria also exist:

(ii.a) **No revelation or partial immediate revelation:** all sellers (and hence buyers) behave as in the monopoly equilibrium.

(ii.b) **Full revelation but with delay:** $q_S(t) \in (0, 1)$ and $q_B(t) \in (0, 1)$ for $1 \leq t < T$, and $q_S(t) = 0$, $q_B(t) = 1$ for $t \geq T$.

(iii) **Asymptotically, full and immediate revelation in all equilibria:** for n sufficiently large all the equilibria in (ii) vanish and the unique equilibrium is the one described in (i).

Proof of Proposition 2: Claim (i) follows immediately from the non-exclusivity of each seller's information. Effectively, if a seller anticipates that his competitors offer price p_L in state L (i.e. choose to fully reveal their information), the unique best response for him is to do the same. The alternative is in fact losing entirely his market share.

Consider next the seller's strategy in the monopoly equilibrium (ii.a). The best possible deviation for a seller, when every other seller follows this strategy, is to announce p_L at a node where the strategy prescribes to offer p_H with probability

1. If the seller undercuts and offers p_L , he sells to the whole market at the price p_L , so that his profits (starting from that node) are $p_L - c_L$. On the other hand, the profits obtained by adhering to the collusive strategy are $\frac{p_H - c_L}{n}$ (i.e. the seller's share of the monopoly profits)²³. Hence, the choice of the monopoly strategy for all sellers remains optimal if and only if $\frac{p_H - c_L}{n} \geq p_L - c_L$, which holds for small enough n ; this proves claim (ii.a).

Turning to (ii.b), the optimality of the prescribed strategy for sellers require them to be indifferent, in each of the first T periods, between offering p_H or p_L . In particular, for $t = T$, we must have:

$$q_S^{n-1}(T)[q_B(T)\frac{p_H - c_L}{n} + (1 - q_B(T))\delta\frac{p_L - c_L}{n}] = q_S^{n-1}(T)(p_L - c_L) + R(T), \quad (4.1)$$

where on the left hand side is the payoff from offering p_H (at T), on the right hand side the payoff from p_L and $R(T)$ denotes the expected payoff to charging p_L in the event that some of the competitors also charge p_L . Since $R(T) \geq 0$, when n is sufficiently large (4.1) cannot hold, no matter what are $q_S(T), q_B(T) \in [0, 1]$, thus showing that even this strategy cannot be part of an equilibrium. On the other hand, for n small (but greater than 2) we may be able to find $q_S(t), q_B(t) \in [0, 1]$ satisfying (4.1) and the analogous equalities for $t = 1, \dots, T - 1$, thus showing that temporary collusion may be an equilibrium in that case.

To complete the proof of the Proposition, we have to show that no other equilibrium exists. Claims (i) and (ii.a) characterize the equilibria where sellers follow pure strategies or randomize for finitely many periods and switch then to p_H ; claim (ii.b) describes the possible equilibria where sellers randomize for finitely many periods and then switch to p_L . It remains to consider the case where sellers in state L randomize for infinitely many periods. By a similar argument to the monopoly case we can show:

Lemma 6. *There is no equilibrium where the sellers in state L randomize for infinitely many periods.*

Proof of Lemma 6: Note that for sellers to randomize at any period t ($0 < q_S(t) < 1$), buyers have to accept p_H with positive probability: $q_B(t) > 0$. Moreover, for the game not to end with probability 1 in finite time, it must be $q_B(t) < 1$ for all t ,

²³Strictly speaking, in the special case where $\alpha_0 = \bar{\alpha}$ the profits are less or equal this level.

i.e. buyers have also to randomize for infinitely many periods. Recall then Lemma 3: this result is still valid and implies that $\lim_{t \rightarrow \infty} \alpha_t = \bar{\alpha}$. But, if sellers randomize during infinitely-many periods, the sequence $\{\alpha_t\}_{t \geq 1}$ is strictly increasing so that, for any t , we have $\alpha_t < \bar{\alpha}$, which contradicts the fact that $q_B(t) > 0$. ■

This completes the proof of the Proposition. ■

4.2. One Informed Oligopolist

We explore a different information structure, in which there is only one informed seller, whose information is then exclusive. As a consequence, the informed seller has the same ability as the monopolist to hide his information. However, to be able to profit from this in the presence of other, uninformed sellers, the informed seller must be able to successfully manipulate both the buyers' and the other sellers' beliefs to induce all of them to trade at a high price, and this may not always be possible, as we will see. Furthermore, even if it were possible we will show that it is not optimal, when the number of sellers in the market is sufficiently large: in that case in fact the incentives to undercut and steal all his competitors' market share prove too strong, even though by so doing all information would be revealed.

More specifically, we will show that, for n small, we have equilibria where no information is revealed (and there may even be no equilibrium with full revelation): the informed seller hides his information, partially or completely, and with some positive probability trade takes place at p_H in both states. However, for n sufficiently large, the equilibrium is unique and reveals immediately the information to the buyers, as in the case where all sellers are equally informed.²⁴

Combining the results of both subsections it becomes apparent that the large number of sellers competing in the market (and the absence of clienteles), but not the differences in the information structure seem to be the key feature to allow information to be fully and immediately revealed to the consumers.

To characterize the equilibria formally, we need now to describe separately both the strategy of the informed seller, in states H and L , and the uninformed sellers (and buyers). As before, the informed seller in state H will always charge p_H . We denote then by $q_I(t)$ and $q_U(t)$ the probability that the state L informed seller and each uninformed seller, respectively, charge p_H at date t . Let $q_B(t)$ be

²⁴We will assume that the identity of the informed seller is known. When this is not known the inferences of the uninformed traders from the observation of the proposals made are more difficult; however, it is possible to show that the validity of our results extends to that case.

the probability that buyers accept p_H in period t , following a history where the only price announced by sellers has been p_H .

Some further notation is also needed to identify the relevant cutoff values in the beliefs of uninformed sellers. Let:

$\tilde{\alpha}$ be the belief that makes uninformed sellers indifferent between trading at p_L and not trading at all: $\tilde{\alpha}(p_L - c_H) + (1 - \tilde{\alpha})(p_L - c_L) = 0$. Thus for $\alpha < \tilde{\alpha}$ an uninformed seller strictly prefers trade at p_L to no trade.

$\hat{\alpha}$ be the belief that makes uninformed sellers indifferent between not trading at all and announcing p_L , when every other uninformed seller proposes p_H while the informed state L seller proposes p_L : $\hat{\alpha}(p_L - c_H) + (1 - \hat{\alpha})\frac{p_L - c_L}{2} = 0$. For $\alpha > \hat{\alpha}$ no uninformed seller would offer p_L in this situation.

It can be easily verified that $\tilde{\alpha} > \hat{\alpha}$.

Proposition 3. *In the model with n sellers, only one of whom is informed, without clienteles:*

(i) *For n large enough, there is always an equilibrium with full and immediate information revelation. Specifically,*

(i.a) **Full information revelation to all uninformed traders:** *for large enough n and for all $\alpha_0 \geq \hat{\alpha}$, there exists an equilibrium where $q_U(1) = 1$, $q_I(1) = 0$ and $q_B(1) = 1$ (that is, at $t = 1$ the uninformed sellers charge p_H , the informed seller charges p_H in state H and p_L in state L , and buyers accept both p_L and p_H , provided all sellers offered p_H).*

(i.b) **Full information revelation to buyers:** *for large enough n and for all $\alpha_0 < \tilde{\alpha}$, there exists an equilibrium where $q_U(1) = q_I(1) = q_B(1) = 0$ (at $t = 1$ the uninformed sellers charge p_L , the informed seller charges p_H in state H and p_L in state L , and buyers accept only p_L at $t = 1$).*

(ii) **No full revelation:** *for n sufficiently small there are equilibria where $q_I(t), q_U(t) \in (0, 1]$ and $q_B(t) \in (0, 1]$ for all $t \leq T$, for some finite $T \geq 1$.*

(iii) **Asymptotically, full and immediate revelation in all equilibria:** *as $n \rightarrow \infty$, the only equilibria are the ones in (i).*

Proof of Proposition 3: (i.a) Given the strategies for the other players described in the statement of (i.a), when $\alpha_0 \geq \hat{\alpha}$ the expected profit for an uninformed seller from offering p_L is non-positive. Hence in this situation an optimal choice for any uninformed seller is indeed to offer p_H , which yields a positive expected profit,

$\frac{1}{n}(p_H - c_H)\alpha_0$. The informed seller in state L then strictly prefers to charge price p_L if:

$$p_L - c_L > \frac{1}{n}(p_H - c_L)$$

always satisfied for n sufficiently high.

(i.b) When the other uninformed sellers, as well as the informed one in state L , offer p_L , an uninformed seller also prefers to charge p_L if:

$$\alpha_0(p_L - c_H)\frac{1}{n-1} + (1 - \alpha_0)(p_L - c_L)\frac{1}{n} \geq 0.$$

If $\alpha_0 < \tilde{\alpha}$ this inequality is always satisfied for n large enough. It is then immediate to see that the informed seller's choice of offering p_L (in state L) is an optimal response to the uninformed's strategy, since $\frac{p_L - c_L}{n} \geq 0$.

(ii) Let n be sufficiently small so that $p_L - c_L \leq \frac{1}{n}(p_H - c_L)$. Note that $p_L - c_L \geq (p_L - c_H)\alpha_0 + (1 - \alpha_0)(p_L - c_L)$ for all α_0 ; the unit expected payoff of trading at p_L is always higher for the informed than for the uninformed seller since the first one can choose to trade at this price only in state L .

First, we have equilibria that resemble the ones found in the monopoly section. That is, if $\alpha_0 \geq \bar{\alpha}$, offering p_H every period both for the uninformed sellers and the informed seller in the H and L states constitutes an equilibrium.²⁵

In addition, there are equilibria that involve randomization on the part of the informed seller in state L for $T > 1$ periods: in period $T + 1$, the informed seller in state L must charge p_L (this low price in the final trading date is needed to sustain the randomization of buyers, which in turn is required for the informed seller to be willing to randomize). In the first T periods, in some of these equilibria the uninformed sellers charge p_H while in others they randomize between p_H and p_L ; in period $T + 1$ they will charge p_L (if $\alpha_{T+1} < \tilde{\alpha}$) or randomize.

(iii) We show finally that as $n \rightarrow \infty$ the only equilibria are those with full and immediate separation described in (i). Evidently, there is no equilibrium where the informed seller in state L charges p_H with probability 1 (as the first of the equilibria described in (ii)): with n large undercutting is always preferred. By an argument similar to the one in the proof of Lemma 6, sellers cannot randomize for infinite periods. We are left then with examining the other equilibria described in (ii), where the type L seller randomizes for T periods before choosing p_L with probability 1.

²⁵If, on the other hand $\alpha_0 < \bar{\alpha}$, there is an equilibrium where the informed seller in state L randomizes in the initial period to induce the belief $\alpha_1 = \bar{\alpha}$.

Suppose first that the uninformed sellers choose p_H at $T + 1$. Then, the indifference condition for the informed seller in period T between p_L and p_H is:

$$q_U^{n-1}(T)(p_L - c_L) + \sum_{r=1}^{n-1} (1 - q_U(T))^r q_U^{n-1-r}(T) \frac{(n-1)!}{r!(n-1-r)!} \frac{p_L - c_L}{r+1} = q_U^{n-1}(T) \left[\frac{p_H - c_L}{n} q_B(T) + \delta(p_L - c_L) \right].$$

For n sufficiently large, the term on the right hand side is approximately $q_U^{n-1}(T)\delta(p_L - c_L)$. This is smaller than the first term on the left hand side because $\delta < 1$, and in addition, the rest of terms on the left hand side are not negligible. Thus, the expression on the left hand side exceeds the one on the right hand side, which is a contradiction.

The same is true, a fortiori, if the uninformed choose p_L with positive probability at $T + 1$. So we conclude that these also cease to be equilibria for n large. ■

The uninformed sellers' behavior also depends on their beliefs about the state. However, in the situation considered here there is still no need to specify off-equilibrium path beliefs after unilateral deviations (of the informed seller), as in the case of monopoly. The inference both of buyers and uninformed sellers is in fact irrelevant when the informed seller (or, for that matter, any seller) announces p_L since the optimal response of buyers is always to accept in this case and hence the game ends immediately. On the other hand, there is no off-equilibrium deviation to p_H .²⁶

In this model information is monopolized by one seller. However, Proposition 3 makes it clear that, as n grows large, the intensity of price competition among sellers in the model with no clienteles gives too strong an incentive to undercut and hence all information is revealed to the buyers right away. The equilibrium in (i.b), that also survives for all n , is also characterized by the fact that all information is immediately revealed to the buyers, but there is no revelation to the uninformed sellers, who end up trading at p_L in state H . Their prior belief giving low probability to the state being high leads them to bear a payoff in that state which is not ex post individually rational.

5. Oligopoly with Clienteles

We analyze here to what extent the results obtained in the previous section (in particular, the fact that all information, whether exclusive or not, is immediately

²⁶With regard to deviations by uninformed sellers, when $n > 2$ there will always be at least one uninformed offering p_L , thus leading to an immediate termination of the game.

revealed to the buyers when there are sufficiently many sellers), remain valid when less extreme forms of price competition are considered. To this end, as anticipated in Section 2, we introduce the model with clienteles, whose main difference from the previous one is that each period a buyer can only buy from his designated seller; hence the term clientele. This association only lasts one period, as each time t the buyers remaining in the market are randomly reassigned to sellers. By undercutting each seller can now steal only a limited fraction of his competitors' market share, as in models of Bertrand competition with capacity constraints or models of Cournot competition.

Again we proceed to study the model when there are n sellers under the two extreme information structures.

5.1. All Informed Oligopolists

If all other informed sellers choose p_L , i.e. to reveal their information, and this is commonly observed by all buyers, the best reply of a seller is clearly to do the same, as long as there is some, even very weak, competition among sellers. Thus full and immediate revelation of the information remains an equilibrium in the model with clienteles, because of the non-exclusivity of the information.

On the other hand, as already mentioned, the gains from undercutting are now much more limited. To see this more precisely, consider the situation where, at some date t in state L all sellers announce p_H . We can construct now different equilibria supporting this outcome using different off-equilibrium beliefs when one seller deviates to p_L . If these beliefs are that the state is L with probability 1, buyers will reject all offers of p_H . Thus only the seller who announced p_L will sell to the $1/n - th$ of the market constituting his clientele for the period. The remaining $(n-1)/n - th$ of the buyers on the market at t will then still be on the market at $t+1$ and will be equally split among the n sellers. At $t+1$ all sellers will offer p_L if the buyers' strategy is to keep rejecting all offers of p_H . Thus by undercutting, an informed seller can only increase his market share from $\frac{1}{n} - th$ to $\frac{1+(n-1)/n}{n} - th$ of the market; moreover, his increase in market share will take one period to materialize. As a consequence, collusive behavior among sellers, and in particular to hide the information and profitably manipulate buyers' beliefs, is now easier. This shows that if the collusive payoff in state L , $(p_H - c_L)$, is tempting enough, the "local monopoly" power created by the presence of clienteles suffices for the existence of another (collusive) equilibrium regardless of the number of sellers: each seller behaves as in the monopoly equilibrium and no information, or only

the minimal amount which is necessary to trade at the high price, is revealed. Furthermore, with other off-equilibrium beliefs, there is a collusive equilibrium that does not necessitate the assumption of the collusive payoff being large enough. We give its details in part (ii) of the next proposition.

This stands in clear contrast with our findings for the model without clienteles. It reveals that, even in the absence of exclusivity of information, no revelation may occur at equilibrium, whatever the number of sellers, when the intensity of competition among sellers is not too strong. It also shows that the nature of competition among sellers appears to play a more important role than the exclusivity of information in determining whether or not information is fully revealed to the buyers, when there are many sellers.

Formally, we have²⁷:

Proposition 4. *In the model with n informed sellers with clienteles:*

(i) **Full and immediate information revelation:** *for all $n \geq 2$ and all α_0 there is always an equilibrium where $q_S(1) = 0$ and buyers accept p_H in period 1 when all prices announced in that period are p_H .*

ii) *For any $n \geq 2$, the following collusive equilibria (where sellers behave as in the case of monopoly) also exist:*

(ii.a) **No revelation:** *when $\alpha_0 \geq \bar{\alpha}$, $q_S(t) = 1$ for all t and buyers immediately accept p_H ;*

(ii.b) **Partial revelation:** *when $\alpha_0 < \bar{\alpha}$ and*

$$p_H - c_L \geq (p_L - c_L) \left[\delta + \frac{(1 - \alpha_0)\bar{\alpha}}{\alpha_0(1 - \bar{\alpha})} \right],$$

$q_S(1) \in (0, 1)$ so that $\alpha_1 \geq \bar{\alpha}$ and buyers accept with probability 1 if all sellers announce p_H ; for $t > 1$ we have $q_S(t) = 1$ and all buyers accept p_H .

Proof of Proposition 4: (i) We omit this proof, as it is similar to that of claim (i) of Proposition 2.

(ii.a) When the strategy of the buyers is to accept p_H if all sellers propose p_H , and that of all the other sellers is to propose p_H in state L at date 1, the profit for a seller if he does the same (offer p_H) is $\frac{p_H - c_L}{n}$. On the other hand, if he were

²⁷We use here again $q_S(t)$ to denote the probability that a seller proposes p_H in state L at date t .

to undercut and charge p_L , he would sell immediately to the $\frac{1}{n}$ share of buyers constituting his clientele at $t = 1$. Assigning off-equilibrium path beliefs after this deviation equal to the beliefs on the equilibrium path and assuming that the sellers' strategy is still to offer p_H at any later date (i.e., the same equilibrium behavior as in period 1), buyers will continue to accept all offers of p_H at $t = 1$. The payoff for undercutting is then only $\frac{p_L - c_L}{n}$, so that charging p_H is clearly the sellers' best response²⁸. Given the sellers' strategy, since $\alpha_0 \geq \bar{\alpha}$, all buyers then prefer to immediately accept both p_L and p_H .

(ii.b) Taking as given the strategies of the buyers and the other sellers as described in part (ii.b) of the statement, the payoff to a seller in period 1 in the event that $r > 0$ sellers other than himself announce p_L is:

$$\delta(p_L - c_L) \frac{n - r}{n^2} \quad (5.1)$$

if he charges p_H , and

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L) \frac{n - 1 - r}{n^2} \quad (5.2)$$

if he charges p_L . The probability of this event is then $(q_S(1))^{n-1-r} (1 - q_S(1))^r \frac{(n-1)!}{(r!(n-1-r)!)}$.

On the other hand, if $r = 0$ sellers other than him charge p_L , his payoff is

$$\frac{p_H - c_L}{n}$$

if he charges p_H ,

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L) \frac{n - 1}{n^2}$$

if he charges p_L and the probability of this event is $(q_S(1))^{n-1}$.

To sustain indifference, we need that the expected payoff of p_H equals the expected payoff of p_L , i.e. that the sum of the above terms describing the payoff associated to p_H , weighted by their respective probabilities, over all r running between 0 and $n - 1$, equals the sum of the corresponding terms describing the payoff associated to p_L . Noting that, for all $r > 0$ the difference between (5.1) and (5.2) equals $\frac{p_L - c_L}{n} [\frac{\delta}{n} - 1]$, simplifying terms we obtain the following equality:

$$(1 - (q_S(1))^{n-1}) \frac{p_L - c_L}{n} [\frac{\delta}{n} - 1] + (q_S(1))^{n-1} \frac{1}{n} [(p_H - c_L) - (p_L - c_L)(1 + \delta \frac{n-1}{n})] = 0,$$

²⁸On the other hand, when the off-equilibrium path beliefs following a deviation to p_L are such that the probability of L is 1 (as discussed earlier, in the second paragraph of this section), the same conclusion holds under the condition $p_H - c_L \geq (p_L - c_L)(1 + \delta)$.

which can be simplified to:

$$\frac{p_L - c_L}{n} \left[\frac{\delta}{n} - 1 - (q_S(1))^{n-1} \delta \right] + \frac{p_H - c_L}{n} (q_S(1))^{n-1} = 0. \quad (5.3)$$

Next, let \bar{q}_S be the value of $q_S(1)$ that generates an updated belief of the buyers, after observing all sellers announcing p_H , of $\alpha_1 = \bar{\alpha}$:

$$\bar{q}_S^n = \frac{\alpha_0(1 - \bar{\alpha})}{(1 - \alpha_0)\bar{\alpha}}.$$

Observe that $q_S(1)$ can be set to take any value between \bar{q}_S and 0, thus inducing a belief $\alpha_1 \geq \bar{\alpha}$, and hence supporting the buyers' choice to immediately accept both p_H and p_L . When $q_S(1) = 0$, the term on the left hand side of (5.3) is clearly negative. On the other hand, when $q_S(1)$ is such that $q_S(1) = \bar{q}_S$, the sign of this term is equal to the sign of:

$$(p_H - c_L) \left(\frac{\alpha_0(1 - \bar{\alpha})}{(1 - \alpha_0)\bar{\alpha}} \right)^{(n-1)/n} + (p_L - c_L) \left[\frac{\delta}{n} - 1 - \left(\frac{\alpha_0(1 - \bar{\alpha})}{(1 - \alpha_0)\bar{\alpha}} \right)^{(n-1)/n} \delta \right] \quad (5.4)$$

which is positive, for all n , under the condition in the statement of part (ii.b)²⁹. Therefore, it is always possible to find a value of $q_S(1) \in [0, \bar{q}_S]$ so that (5.3) is satisfied. ■

5.2. One Informed Oligopolist

As we saw in the previous section, in the presence of clientele hiding the information can be profitable for an informed seller, whatever the number of sellers (this is true in particular when the profits per unit sold at a high price are sufficiently higher than the profits from a sale at a low price). However, with non-exclusive information a fully revealing equilibrium always exists, together with collusive equilibria, for any n . When there is only one informed seller, information is exclusive and, as already argued in Section 4.2, the informed seller can always hide it,

²⁹The term on the left hand side of (5.4) is positive if $(p_H - c_L) > (p_L - c_L) \left[\delta + \left(1 - \frac{\delta}{n}\right) \left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})} \right)^{(n-1)/n} \delta \right]$. Noting that $\left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})} \right)$ is the reciprocal of a probability, and hence is greater than 1, we have $\left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})} \right) > \left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})} \right)^{(n-1)/n}$ so that the condition in the statement of (ii.b) can be used in the above inequality to get the result.

as in the case of monopoly. We should expect then that in this situation full revelation is difficult to achieve. Indeed, we will show that, for many configurations of parameters describing an economy, separation is impossible at equilibrium and an equilibrium exists where at least part of the information is never revealed. The only case in which hiding the information can prove too costly for the informed seller is when both the gain in the per unit profit obtained by selling at p_H in state L is sufficiently low and the prior belief of the uninformed sellers sufficiently optimistic so they may be willing to offer a low price.

The results are again in clear contrast with what we found in the model without clienteles. If information is exclusive, we should not expect it to be revealed at equilibrium, when the intensity of the competition among sellers is not too strong.

Proposition 5. *In the model with n sellers, only one of whom is informed, and with clienteles:*

(i.a) **Full revelation is impossible for many parameter configurations:** for any $n \geq 2$, if $p_H - c_L > (p_L - c_L)(1 + \delta)$ (the collusive payoff is not too small), or $\alpha_0 < (p_L - c_L) / (c_H - c_L)$ for all t (beliefs are sufficiently pessimistic), at equilibrium we have $q_I(t) > 0$ for all t ;

(i.b) **Full revelation only occurs for some parameter values:** there exists n large enough such that, for $p_H - c_L < (p_L - c_L)(1 + \delta)$ and $\alpha_0 > (p_L - c_L) / (c_H - c_L)$, there is an equilibrium with $q_I(1) = 0$.

(ii) For any $n \geq 2$, the following additional equilibria exist:

(ii.a) **No revelation:** when $\alpha_0 \geq \bar{\alpha}$, $q_U(t) = q_I(t) = 1$ for all t (both the uninformed sellers and the informed seller in both states charge p_H) and buyers immediately accept;

(ii.b) **Partial revelation:** when $\alpha_0 < \bar{\alpha}$ and $(p_H - c_L) > (p_L - c_L)(1 + \delta)$, $q_I(1) \in (0, 1)$ and $q_U(1) = 1$ (the informed seller randomizes in state L in the initial period, while the uninformed charge p_H); for all $t > 1$ we have $q_I(t) = q_U(t) = 1$. In every period, buyers randomize between accepting and rejecting when all announced prices are p_H .

Proof of Proposition 5: (i.a) Suppose full separation occurs in equilibrium in some period t . Then, the state H informed seller charges p_H in that period, and the state L informed seller charges p_L . By Bayes' rule, the buyers and the uninformed sellers must update their beliefs at t to $\alpha_t = 1$, upon observing the informed seller charging p_H , and to $\alpha_t = 0$ upon observing the informed seller charging p_L . If p_L is

observed, the clientele of the informed seller must clearly accept. But so must the clientele of the informed seller if he charges p_H , because the only reason to reject would be to expect p_L in the future. However, the low price in the future could only come from the uninformed sellers, who believe now that the state is H with probability 1, and hence will never charge the low price. Therefore, the clientele of the informed seller must accept both p_H and p_L . It follows that in state H all units are sold in period t because all buyers accept even p_H . When all the uninformed announce p_L (which may only occur when $\alpha_0 \leq \alpha_t < (p_L - c_L) / (c_H - c_L)$), the informed seller's profits from announcing p_L in state L are then $\frac{p_L - c_L}{n}$, always smaller than the profits from announcing p_H , given by $\frac{p_H - c_L}{n}$. On the other hand, when all the uninformed announce p_H , the informed's profits from announcing p_L in state L are $\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2}$, again smaller than the profits from announcing p_H ($\frac{p_H - c_L}{n}$) under the conditions stated in (i.a). Thus, under those conditions the informed seller in state L has an incentive to deviate and charge p_H .

(i.b) The strategies supporting separation are as follows. The informed seller charges p_L in state L and p_H in state H in every period. The uninformed sellers also charge p_H in period 1, but switch to p_L if they observe the informed seller charging p_L . Buyers accept both prices if they observe the informed seller charging p_H . They accept only p_L if they observe the informed seller charging p_L .

The optimality of the buyers' strategy easily follows from the degeneracy of their beliefs (obtained by using Bayes' rule at all information nodes where the informed charge p_H or p_L , thus assigning off-equilibrium beliefs when only uninformed sellers deviate equal to the ones on the equilibrium path). Considering next the informed seller in state L , given that the collusive payoff is not too attractive (the first condition on the parameter values imposed in (i.b)), there exists n large enough such that p_L is a best response:

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2} > \frac{p_H - c_L}{n}.$$

Finally, the uninformed sellers prefer to charge p_H at $t = 1$ if

$$\alpha_0 \frac{p_H - c_H}{n} + \delta(1 - \alpha_0) \frac{(p_L - c_L)(n-1)}{n^2} > \alpha_0 \frac{p_L - c_H}{n} + (1 - \alpha_0)(p_L - c_L) \left(\frac{1}{n} + \frac{\delta(n-2)}{n^2} \right),$$

which simplifies to:

$$\alpha_0(p_H - p_L) > (1 - \alpha_0)(p_L - c_L)(1 - \delta/n).$$

The validity of this condition is ensured by the assumption made on α_0 in (i.b) for large enough n .

(ii.a) We construct here an equilibrium where p_H is announced by all sellers in period 1. Therefore, the belief held by uninformed traders is equal to their prior α_0 . Because $\alpha_0 \geq \bar{\alpha}$, buyers are at a best response accepting p_H . Furthermore, we consider the case where the off-equilibrium belief, following a deviation to p_L in period 1, remains α_0 ; hence p_H continues to be accepted by buyers. In this situation, charging p_H is clearly the best response for the informed seller in state L as the effect of charging p_L would be to lower the profits from $\frac{p_H - c_L}{n}$ to $\frac{p_L - c_L}{n}$ (because of the clientele friction, given the assigned off-equilibrium beliefs, undercutting would only have the effect of lowering revenues over the same market share³⁰).

By essentially the same argument, charging p_H is also a best response for any uninformed seller. If he were to deviate to charging p_L , the beliefs held by uninformed traders are unchanged and hence buyers continue to accept p_H from all the other sellers. Thus his expected profits would be $\alpha_0 \frac{p_L - c_H}{n} + (1 - \alpha_0) \frac{p_L - c_L}{n}$, lower than the ones obtained by charging p_H , given by $\alpha_0 \frac{p_H - c_H}{n} + (1 - \alpha_0) \frac{p_H - c_L}{n}$.

(ii.b) Let q_B be the probability with which buyers accept p_H , in any period t , when all prices announced (at t and any earlier date) are p_H . By Bayes' rule, upon observing p_L charged by the informed seller and p_H by all uninformed, the posterior belief is that the state is L with probability 1.

We begin by analyzing the incentives of the informed seller in state L . For him to be willing to randomize in period 1 the following equality must hold:

$$\frac{p_L - c_L}{n} \left[1 + \delta \frac{n-1}{n} \right] = \frac{p_H - c_L}{n} \left[q_B + \frac{\delta(1 - q_B)q_B}{1 - \delta(1 - q_B)} \right] \quad (5.5)$$

Under the condition we imposed that the collusion payoff is attractive enough, it is easy to see that, for fixed δ and n , there exists a unique value of q_B that makes the equality hold. This will be the equilibrium value of q_B . At any later date $t > 1$ the payoff from announcing p_H and p_L are again the same (when the off-equilibrium beliefs, following a deviation to p_L , are that the state is L with probability 1); hence charging p_H is (weakly) a best response.

Consider next the uninformed sellers. If one of them deviates to p_L we assign off-equilibrium beliefs equal to the ones on the equilibrium path. Clearly the payoff associated to this deviation is only positive provided $p_L > \alpha_0 c_H + (1 -$

³⁰As in Proposition 4, when the off-equilibrium path beliefs following a deviation to p_L are, on the other hand, such that the probability of L is 1, a limited increase in the seller's market share is obtained if he undercuts. The deviation is again non profitable under the condition $p_H - c_L \geq (p_L - c_L)(1 + \delta)$.

$\alpha_0)c_L$. In that case, if the strategy of all traders following this deviation prescribes offering p_L , it is easily verified that the expected payoff for deviating to p_L is $\frac{p_L - (\alpha_0 c_H + (1 - \alpha_0)c_L)}{n} [1 + \delta \frac{n-1}{n}]$. This expression, taking into account (5.5), is strictly smaller than the expected payoff from p_H , $\frac{p_H - (\alpha_0 c_H + (1 - \alpha_0)c_L)}{n} [q_B + \frac{\delta(1 - q_B)q_B}{1 - \delta(1 - q_B)}]$. This shows that the deviation considered is unprofitable.

Note, finally, that the probability that the informed seller charges p_H at $t = 1$ is chosen so as to yield $\alpha_1 = \bar{\alpha}$. Thus, buyers are willing to randomize at $t = 1$ between accepting and rejecting upon the observation of all prices being p_H . Given that pooling on p_H takes place from period 2 on, buyers are best-responding by continuing their randomization in any period $t > 1$. ■

Thus, for a large subset of the parameter region full separation never occurs and only collusive equilibria exist, whatever the number of sellers n . For the complementary region, separation can be supported, but collusive equilibria are also found. Thus, the message of the model with clienteles is somewhere between the positive findings - with regard to information revelation and hence convergence to REE - of the model without clienteles, where competition is quite intense, and the negative results of the pairwise meetings literature, where the local monopoly power of each seller in each meeting is reinforced by the lack of observability of public signals that could help reveal the information (see Wolinsky (1990), Blouin and Serrano (2001)).

6. Discussion

We conclude by discussing the robustness and efficiency properties of the equilibria with $n > 1$ sellers found in the two previous sections. The analysis can build, to a large extent, on the properties derived in this regard for the monopoly case and hence we will just concentrate on the elements of difference.

In particular, similar comments can be made concerning the robustness of the equilibria of the oligopoly models to the relaxation of the restriction that sellers can only propose one of two given prices. The full and immediate revelation result of the model without clienteles for n large enough is again found with a continuum of possible prices, though now there will be a continuum of such equilibria. Analogously, the model with clienteles has equilibria with no or only partial information revelation, when $n \rightarrow \infty$.

6.1. Welfare

6.1.1. Oligopoly without clienteles

- a) When all sellers are informed, the economy is one of non-exclusive information; in particular, every interim individually rational allocation can be made incentive compatible by punishing contradictory reports about the state with the “no trade” outcome. In this case the only equilibrium that survives for any number of sellers n is the one with full information revelation. As we argued, this is not only ex post but also interim Pareto efficient because of the absence of delay. Recall that traders are risk neutral, which implies that the interim gains from trade are just the appropriately weighted averages of ex post surpluses. In this equilibrium each seller in state H receives a payoff of $(p_H - c_H)/n$, each seller in state L a payoff of $(p_L - c_L)/n$, and buyers get an expected payoff of $\alpha_0(u_H - p_H) + (1 - \alpha_0)(u_L - p_L)$.³¹
- b) When only one seller is informed, information is exclusive. Two types of equilibria exist for all n . Both are separating equilibria and the information, privately held by the only informed seller, is fully revealed to the buyers.
- b1) In the equilibrium described in part (i.a) of Proposition 3, the informed seller in state H receives a payoff of $(p_H - c_H)/n$, while in state L manages to steal all the market from the other sellers and receives a payoff of $p_L - c_L$. Uninformed sellers receive an expected payoff of $\alpha_0(p_H - c_H)/n$. Each buyer continues to receive the same expected payoff as when all sellers are informed, $\alpha_0(u_H - p_H) + (1 - \alpha_0)(u_L - p_L)$. This equilibrium is also interim efficient and the total payoff of sellers and buyers is the same as with all sellers informed. The only difference one should stress is the transfer of surplus from the uninformed sellers to the informed seller in state L .
- b2) While the equilibrium described in part (i.b) of Proposition 3 is also interim incentive efficient because of the absence of delay in trading, its features are somewhat distinct. In this equilibrium the more optimistic beliefs of the uninformed traders crowd out the informed seller in state H , who receives a payoff of 0. The informed seller in state L receives a payoff of $(p_L - c_L)/n$, and the uninformed sellers get an expected payoff equal to $\alpha_0(p_L - c_H)/(n - 1) + (1 - \alpha_0)(p_L - c_L)/n$. Buyers are of course the clear winners, receiving

³¹When compared to the pooling equilibrium obtained under monopoly, the sum of the payoffs of sellers in state L is now lower while buyers' payoffs are higher.

an expected payoff of $\alpha_0 u_H + (1 - \alpha_0)u_L - p_L$. Recall that even though this equilibrium is interim efficient, information is not revealed to the uninformed sellers, all trade occurs at p_L in state H and uninformed sellers, driven by their optimistic beliefs, end up transacting at a price that is not ex post individually rational (in H).

The equilibrium in Proposition 3, part (i.b), constitutes a significant exception to the identification of interim incentive efficiency and full, immediate information revelation.

6.1.2. Oligopoly with clientele

With regard to the case of all informed sellers, the allocation at the separating equilibrium of Proposition 4, part (i), is the same as the one in the equilibrium found in Proposition 2, for n large. The other collusive equilibria obtained in Proposition 4 are very close to the ones found in Proposition 1 for the case of the monopolist. The reader is referred to the previous sections for discussions of their welfare properties.

Consider next the equilibria with only one informed seller. When separation is sustained in equilibrium all trade takes place at p_L in state L and at p_H in state H . The welfare properties of this equilibrium are similar to the ones of the separating equilibrium described in (ia) of Proposition 3 (with two differences: (i) there is delay, though this should now be viewed as a trade friction generated by the presence of clientele; (ii) the distribution between uninformed and informed of the sellers' profits in state L is now more equal).

In contrast, at the equilibria where the informed hides his information, the total payoff of buyers and sellers is the same as at the equilibria obtained under monopoly. In some of these equilibria, delay occurs due to the buyers' randomization and this causes inefficiencies. The same incentive compatible mechanism proposed earlier (Section 3.1) in the monopoly case allows to find an improvement in this case: the informed seller in state L , rather than randomizing charges p_L with probability 1, while buyers, when all prices announced are p_H , keep randomizing according to the same probabilities as in the equilibrium. Using the same arguments as in Section 3.1, one can see that this mechanism, which is incentive compatible, interim dominates the equilibrium. Note also that the expected payoff for the informed and uninformed sellers is the same in this case (unlike in the equilibria of the model without clientele); thus the existence of clientele exerts

a positive externality on the uninformed sellers and, clearly a negative externality on buyers, whose payoff is lower.

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