A Credit Market à la David Hume

Menahem E. Yaari[*]

1. Introduction

In Book III of his *Treatise of Human Nature*, David Hume considers the following simple interaction:

> I suppose a person to have lent me a sum of money, on condition that it be restor'd in a few days; and also suppose, that after the expiration of the term agreed on, he demands the sum

and Hume asks:

> *What reason or motive have I to restore the money*? [1740, p.479]

The answer, according to Hume, could be the borrower's "abhorrence of villainy and knavery", but this could only be the case in "a civiliz'd state, when [the borrower] is train'd up according to a certain discipline and education". This cannot be the answer, however, when the interaction is looked at from the perspective of man's basic "rude and more *natural* condition". Nor can the answer be found in "a concern for [the borrower's] private interest or reputation" (repeated game considerations) or in "a regard to the interest of the [lender]" (altruism). The analysis of certain specific scenarios (supposing, for example, "that the loan was secret" or that the lender "be a vicious man") enables Hume to show that these answers, too, cannot be right. The answer, he therefore concludes, must be "that the sense of justice and injustice [which is the motive for repaying the loan] is not deriv'd from nature, but arises artificially, tho' necessarily, from education and human conventions" (p. 483).
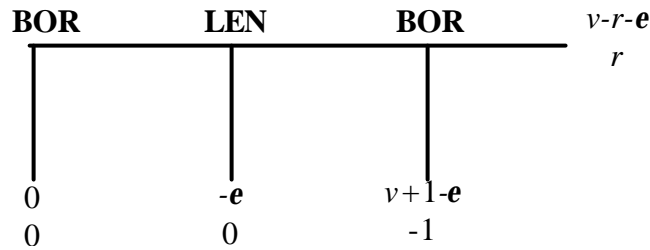
It is important to appreciate the meaning of the terms "natural" and "artificial", as used by Hume. The former refers to traits and norms that correspond to man's primary motives (such as "self-love, when it acts at its liberty") whereas the latter refers to traits and norms that apply to "man in his civiliz'd state". Thus, the term "natural" describes the modes of behavior and interaction that characterize Hobbes's "State of Nature", whereas "artificial" describes the modes of behavior and interaction that characterize a "civilized" society. The transition from the State of Nature to a civilized society is thought of as an **evolutionary** process whereby, mainly through education, certain **conventions** take hold and thrive. Hume's perspective is clearly evolutionary in essence, with the term "artificial" (which, Hume emphasizes, is not to be understood in the sense of "arbitrary" or "contrived") being used to describe a kind of mutation-driven drift from the State of Nature to higher forms of interaction. The agents through which the emerging modes of behavior are passed on from generation to generation (Dawkins's "memes" ?) are the conventions. Repaying one's loans, even when issues such a retribution and long-term reputation do not arise, is an example of such a convention, and Hume obviously thinks, moreover, that it is a convention capable of long-term persistence.

It is my purpose in this essay to offer formal (and modern) underpinnings for Hume's argument. I shall do so in the context of Hume's own example, cited above, where the interaction being considered is one between lender and borrower.

There have been several attempts, in the literature, to provide a modern version of David Hume's conventions, notably those of David Lewis [1967] and Peyton Young [1993]. The focus in these studies was an aspect of David Hume's conventions which is different from the one to be explored herein. Specifically, Hume sees conventions also as instruments of **coordination**, with language being the most important case in point. Coordination, notably coordination towards one specific equilibrium among the possibly many that exist, is the feature of most recent studies of conventions. Here, conventions will be studied in their role as vehicles for supporting seemingly non-self-seeking behavior, such as repaying one's loans when self-interest would indicate otherwise.

2. A Simple Credit Market

Let us think of a credit market where n potential borrowers face one lender. The case of a single borrower ($n = 1$) will be as in David Hume's above cited vignette, namely:

That is,

- The borrower must decide, first of all, whether or not to seek a loan from the lender.

- Seeking the loan is in itself costly, requiring the payment of an "application fee" e ($e > 0$).

- The loan application can either be **approved** or be **rejected** by the lender.

- The rate of interest being charged, on a standard loan of size 1, is r.

- The value of the loan to the borrower is denoted v. It is assumed that $v > r+e$, i.e., obtaining the loan is worthwhile.

- If the loan application is approved, the borrower can either **repay** (with interest) or **renege** (**default**).

In extending this basic interaction to the case of n potential borrowers, we shall keep matters simple by assuming that –

- The decision regarding whether or not to seek a loan is made by each potential borrower independently of the decisions of the others.

- All loans are identical, both with regard to size (normalized at 1) and with regard to the rate of interest being charged, r.

- The value v of a loan is common to all potential borrowers.

- The lender's action comes after **all** the borrowers have made their initial moves, i.e., after all applications for loans have been filed.

- The players (lender and potential borrowers alike) **do not randomize** their moves**.**

And finally, an assumption is made that explicitly rules out all long-term or repeated game considerations, such as the borrowers' reputations:

- After the interaction, all the borrowers **vanish into the crowd**, never to be recognized again.
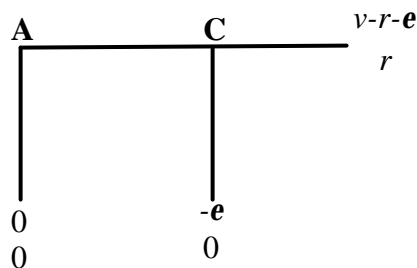
Clearly, regardless of the precise timing of actions, or of the exact definitions of players' strategies (these will be specified shortly) we are going to have the **inactive market** as this system's unique equilibrium point. In other words, the only way for the interaction, as described, to resolve itself coherently is for all borrowers to play "out" in the first move (i.e. to refrain from applying for credit) and for the lender to play "deny all", i.e., to reject all credit applications received, no matter how many (if any) had been filed.

Thomas Hobbes would presumably argue at this point that the rational thing for borrowers to do would be to relinquish their choices and place them in the hands of the Sovereign.
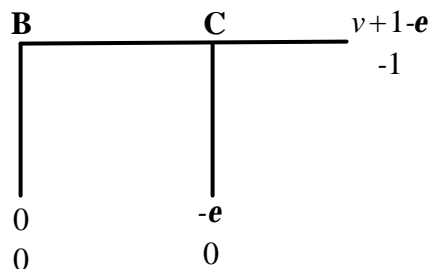
3. Enter David Hume

Now suppose that a **convention**, under which the repaying of loans is thought to be one's duty, somehow gains a foothold among potential borrowers. (We could speak, figuratively, of a loan-repaying **mutant** coming into being.) Thus, there are now **two** types of borrowers in existence, namely **loan repayers** (to be known as borrowers of **Type A**) and **maximizers** (**Type B**). We shall assume the borrower's type (A or B) to be **private information**, so that there does not exist an outwardly observable signal that is in any way informative regarding a given borrower's type.

As before, let there be n potential borrowers and suppose that, among them, k are type A's ($0 \leq k \leq n$) while $n-k$ are Type B's. There is one lender ("The Credit Agency") to whom all potential borrowers must turn when applying for a loan. Let the Credit Agency be denoted C, for short. The lender-borrower game of section 2 now exists in two different versions, depending on the borrower's type. Specifically, when the borrower is of Type A, we have:

and when the borrower is of Type B:

reflecting the fact that A's always repay their loans, while B's never do. (Recall that at the end of the interaction all borrowers vanish into the crowd, never to be personally recognized again.)

We may now consider the following ($n+1$)–player game:

– There are k players of Type A, each with strategy set $\{0, 1\}$.

– There are $n-k$ players of Type B, each with strategy set $\{0, 1\}$.

– The strategy set of the ($n+1$)st player ("Player C") is $\{0, 1, \ldots, n\}$.

The interpretation of the players' strategies is as follows: For any borrower, playing "1" means "apply for a loan", while "0" means "stay out". For player C, playing some integer c, $c \in \{0, 1, \ldots, n\}$, means "grant **all** pending loan applications if their number

does not exceed *c*, and deny **all** pending loan applications otherwise". This definition of Player C's strategies is a spelling out of the following assertions: (1) The Credit Agency moves only after all borrowers had made their moves (i.e., after all loan applications have been filed). (2) The Credit Agency cannot distinguish among pending loan applications, so all it can act on is their number.

The parameters *n* and *k* are assumed to be common knowledge among the players.

Before writing down the players' payoff functions, let us observe that, due to the anonymity of borrowers, any profile of strategies can be fully represented by a triple of integers of the form *(a, b, c)* where:

a is the number of Type A's playing "1".

b is the number of Type B's playing "1".

c is C's strategy (i.e., C's choice from $\{0, 1, \ldots, n\}$).

Given any triple of integers *(a, b, c)* satisfying $0 \leq a \leq k,\ 0 \leq b \leq n\text{-}k,\ 0 \leq c \leq n$, let $Y_i^A(a, b, c)$ be player *i*'s payoff under *(a, b, c)* if *i* is a borrower of Type A, and similarly $Y_i^B(a, b, c)$ for the case where player *i* is a borrower of Type B, with $Y^C(a,b,c)$ being the payoff for player C. Then,

$$Y_i^A(a, b, c) = \begin{cases} (v\text{-}r\text{-}\boldsymbol{e})s_i & \text{if } a+b \pounds c \\ \\ -\boldsymbol{e}s_i & \text{if } a+b > c \end{cases}$$

$$Y_i^B(a, b, c) = \begin{cases} (v+1\text{-}\boldsymbol{e})s_i & \text{if } a+b \pounds c \\ \\ -\boldsymbol{e}s_i & \text{if } a+b > c \end{cases}$$

$$Y^C(a, b, c) = \begin{cases} ar\text{-}b & \text{if } a+b \pounds c \\ \\ 0 & \text{if } a+b > c \end{cases}$$

where s_i is the strategy (0 or 1) actually played by player *i*.

We now have a well-defined game in strategic form, to be referred to in the sequel as the **credit game**.

4. Equilibrium in the Credit Market

Let a, b and c be integers satisfying $0 \leq a \leq k$, $0 \leq b \leq n\text{-}k$, $0 \leq c \leq n$. We shall say that (a, b, c) is an **equilibrium of the credit market** if there exists a Nash equilibrium of the credit game where the number of A's playing "1" is a, the number of B's playing "1" is b, and player C's chosen strategy is c. An equilibrium (a, b, c) of the credit market is said to be **active** if $c > 0$. Finally, an active equilibrium (a, b, c) of the credit market is said to be **maximally active**, if there does not exist an equilibrium $(a\text{¢}\, b\text{¢}\, c\text{¢})$ with $c\text{¢} > c$.

To illustrate these concepts, let us consider the simplest non-trivial example, namely the case $n = 2$, $k = 1$. Let us assume also that $r < 1$. The credit game in this case has 3 players, namely 2 borrowers – an A and a B – plus the Credit Agency C. The game can be written out as follows, with A picking a row, B picking a column and C picking a matrix:

	0	**1**
INACTIVE → **0** (c=0)	0 0 0	0 -*e* 0
1	-*e* 0 0	-*e* -*e* 0

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(c=1)

	0	**1**
0	0 / 0 / 0	0 / $v+1-e$ / 0
1	$v-r-e$ / 0 / r (ACTIVE)	$-e$ / $-e$ / 0

(c=2)

	0	**1**
0	0 / 0 / 0	0 / $v+1-e$ / -1
1	$v-r-e$ / 0 / r	$v-r-e$ / $v+1-e$ / $r-1$

The game obviously has 2 equilibria in pure strategies, namely (0, 0, 0) and (1, 0, 1). Of these, the first is inactive and the second is active. Indeed, (1, 0, 1) is also **maximally** active, since there does not exist an equilibrium with $c = 2$ (since $r < 1$).

The equilibria of the credit market are easily characterized in the general case. Specifically, we have:

Proposition: Let a, b and c be integers satisfying $0 \leq a \leq k$, $0 \leq b \leq n\text{-}k$, $0 \leq c \leq n$. Then *(a, b, c)* is an equilibrium of the credit market if, and only if, the following two conditions hold:

(i) $a + b = c$

(ii) $ar - b \geq 0$

Proof: Condition (i) holds if, and only if, none of the borrowers (players 1, 2, …, n) has a unilateral switch of strategy that would be payoff-improving. Condition (ii) is the same statement for the Credit Agency (player C). ‡

The following remarks follow readily from this Proposition:

(1)The inactive market ($a = b = c = 0$) is always an equilibrium; (2) in any active equilibrium, we must have $a > 0$, i.e., the market cannot be active without the participation of some borrowers of Type A; (3) if *(a, b, c)* is an active equilibrium, then the inequality:

$$\frac{b}{a} \leq r$$

holds, so it is in everybody's interest that the number of Type A players actively participating (i.e., playing "1") be large;   (4) the number of Type A borrowers that must be active in order to support an active borrower of Type B in equilibrium increases as the rate of interest declines.

The foregoing Proposition tells us also that, when $r > 0$ and $k > 0$, there always exists an active equilibrium and, *a-fortiori*, there also exists a **maximally** active equilibrium. Indeed, the maximally active equilibrium is easily characterized, as follows:

For any real number x, let $[x]$ be the greatest integer that does not exceed x. Define $\hat{b}$ by writing:

$$\hat{b} = \min\left(n - k, [rk]\right)$$

Assume that $k > 0$. Then, *the triple (k, $\hat{b}$, $k+\hat{b}$) is an equilibrium of the credit market which, moreover, is maximally active.* This characterization is an immediate consequence of the Proposition.

Let $\hat{Y}^A$, $\hat{Y}^B$ and $\hat{Y}^C$ be the average (*per player*) payoffs of, respectively, a borrower of Type A, a borrower of Type B, and the Credit Agency, C, in the maximally active equilibrium. Then,

$$\hat{Y}^A = v - r - e$$
$$\hat{Y}^B = \frac{\hat{b}(v + 1 - e)}{n - k}$$
$$\hat{Y}^C = rk - \hat{b}$$

We note, finally, that if $[rk] < n\text{-}k$, the equilibrium of the credit market – even when it is maximally active – involves **credit rationing**, in the sense that some potential borrowers who would love to borrow at terms being offered in the market are being barred in equilibrium from carrying out their wish.

5. Evolutionary Dynamics

Imagine now that loan-repaying borrowers (Type A) are pitted against relentless maximizers (Type B) in an evolutionary struggle. Folklore tells us that the Type A's can never prevail. (*Proof:* B's can always choose to mimic whatever the A's are doing, so their reproductive success must always be at least as high as that of A's. Moreover, if the B's – being maximizers – ever choose to deviate from mimicking the A's, the result must be such as to give them reproductive advantage.) This folk tale, pretty though it is, happens also to be false, as will now be shown.

Let us return to the credit market of sections 3 and 4, with one Credit Agency serving n potential borrowers, k of whom A's and $n\text{-}k$ B's. Let the proportion of A's in the borrower population be denoted p ($p = k/n$) and suppose that the evolution of p over time is governed by the relative overall performance of the two types of borrowers. More precisely, we are now allowing the credit market to operate repeatedly with the values of n and k, and even the identities of the players, possibly changing along the

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process. In each period, the credit game of section 4 is played out and, after players receive their payoffs, all the borrowers vanish into the crowd and nobody can be recognized when the market re-convenes in the next period. Indeed, next period's players need not even be the same individuals. (This assumption is made, it will be recalled, in order to rule out repeated game considerations, such as reputation effects.)

We are now ready to describe the manner in which evolutionary pressures affect the progress over time of the population parameter p. Specifically, let us assume that

$$p^{NEXT} \underset{<}{\overset{>}{=}} p \text{ according as } \hat{Y}^A \underset{<}{\overset{>}{=}} \hat{Y}^B$$

where p^{NEXT} stands for the value of p in the next period, and $\hat{Y}^A$, $\hat{Y}^B$ are the per-player payoffs earned in this period by borrowers of Type A and B, respectively. Thus, all that is being asserted is that the **direction** of change in p is such as to reflect the relative success (as measured by average payoff) of the two types. This is a simple instance of the evolutionary rule known as "replicator dynamics".

A **rest point** of the evolutionary process is a value p^* such that at $p = p^*$ we have $p^{NEXT} = p$. A **long–term equilibrium** is defined as a dynamically stable rest point.

Assume, finally, that the credit market proceeds along a path of maximally active within-period equilibria. We could perhaps think of some institution (the Fed?) seeing to it that, in every period, the financial market is as active as it can be, consistent with equilibrium.

Recall that $\hat{b} = \min(n\text{-}k, [rk])$ is the number of Type B borrowers who obtain loans in a maximally active equilibrium. Clearly, if $\hat{b} = n\text{-}k$, i.e., if **all** potential borrowers of Type B become active in the market, then the corresponding value of p cannot be a rest point (when all the B's are in, the average, per player, payoff of a B exceeds that of an A). Turning therefore to values of p for which $b = [rk]$, we find that

$$\hat{Y}^A = v - r - e$$
$$\hat{Y}^B = \frac{[rk]}{n-k}(v + 1 - e)$$

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To locate a rest point, a value of p must be found for which $\hat{Y}^A = \hat{Y}^B$. Using the approximation $[rk] \approx rk$, we find $\hat{Y}^B$ being given by

$$\hat{Y}^B = \frac{rp}{1-p}(v+1-e)$$

which, in turn, yields a unique value for p, call it p^*, for which $\hat{Y}^A = \hat{Y}^B$. Specifically, we find that

$$p^* = \frac{1}{1+r}\frac{v-r-e}{v-e}$$

From the fact that the odds $p/(1-p)$ are increasing in p, it follows immediately that, for values of p satisfying $p \neq p^*$, we have $(p^*-p)(\hat{Y}^A - \hat{Y}^B) > 0$. Hence, p^* is in fact a (unique) long-term equilibrium. Note that, strictly speaking, $p = 0$ and $p = 1$ are also rest points of the dynamics, even though Y^A is not well defined at $p = 0$ (there being no A's) and Y^B is not well defined at $p = 1$ (there being no B's). However, both these rest points are unstable, so p^* is indeed the unique long-term equilibrium.

The fact that $v > r+e$, i.e., that the value of a loan exceeds its cost, ensures that $p^* > 0$. In other words, the long-term survival of loan repaying borrowers (Type A's) is guaranteed. It should be noted also that $p^* \to 1$ as $r \to 0$, reflecting the fact that when the rate of interest is lower, more loan repayers are needed, in equilibrium, to uphold the market.

Finally, looking at the definition of p^*, we see that the inequality $p^* < 1/(1+r)$ always holds. From this inequality it follows that in the long-term equilibrium there are always some borrowers of Type B who are left out of the market in equilibrium. (The inequality $p < 1/(1+r)$ is equivalent to $rk < n-k$ and $[rk] \leq rk$.) In other words, the long-term equilibrium always involves credit rationing. Of course, loans made to those Type B borrowers who do enter the market are going to wind up in default, and the Credit Agency takes this into account. Calculating the ratio of loans made to Type B's to the total of loans granted, we find the **equilibrium default rate** to be given by the quantity $r/(1+r)$.

6. Concluding Remark

In discussions concerning the nature of rationality, it is usually held that maximizing behavior is bound to win out over non-maximizing behavior in the evolutionary struggle. What this essay shows is that if short-term equilibrium is required in every period along the dynamic evolutionary path, then this widely believed assertion may fail, even when reputation and other long-term concerns play no role. The nature of the short-term equilibria may be such as to protect the non-maximizers from eventual extinction and even to allow them to prosper in the long term. The relevance of this conclusion may lie well beyond our analysis of a simple-minded credit market.

7. References

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