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A CONSTRAINT ON THE DISTANCE DEPENDENCE OF THE GRAVITATIONAL CONSTANT

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Extended supergravity theories predict the existence of vector and scalar bosons, besides the graviton, which in the static limit couple to the mass. An example is the graviphoton, leading to antigravity. If these bosons have a small mass ($\lesssim 10^{-4}$ eV), an observable Yukawa term would be present in the gravitational potential in the newtonian limit. This can be parametrized by a distance dependent effective gravitational "constant" $G(r)$. Defining $G_0 = G(10\text{ cm})$ and $G_c = G(10^3\text{ km})$, the comparison between theory and observations of the white dwarf Sirius B results in $G_c/G_0 = 0.98 \pm 0.08$.

The newtonian limit of gravity has only been tested to a high accuracy at laboratory distances and in the solar system at distances of 10^3–10^8 km. Deviations from the $1/r^2$-force law at intermediate distances are not at all excluded, since gravity experiments or observations in this range are very poor [1]. A comparison between astrophysical theory and observations imposes indirect restrictions on possible deviations, as will be shown for the case of the white dwarf Sirius B. But first, as a recent motivation for considering possible non-newtonian terms in the static limit of gravity, supergravity theories will be mentioned briefly.

General relativity is one of the simplest classical relativistic field theories of gravity, and is in agreement with all astronomical observations. Attempts at quantization of general relativity, in the form of a gauge theory with spin 2 gravitons as gauge bosons under the group of general coordinate transformations, showed the theory not to be renormalizable when coupled to matter fields [2]. Introduction of supersymmetry, between fermions and bosons, leads naturally to supergravity as a better candidate for a renormalizable quantum field theory of gravity [3].

Extended supergravity theories exhibit a unique mixing of internal and space–time symmetries, thereby providing a framework for unification of gravity with weak, electromagnetic and strong interactions [3], However, supersymmetry does not seem to be realized at energies available in present accelerators. It seems likely that typical supergravity effects will be clear only at energies of the order of the Planck energy, $E_{\mathrm{P}} = \hbar^{1/2}G^{-1/2}c^{5/2} = 1.2 \times 10^{19}$ GeV. At much lower energies the only detectable gravitational forces are those between matter in bulk. Therefore macroscopic gravity experiments might well form, at least at present, the most direct way to test supergravity theories.

The experimental success of general relativity shows that gravitation can be described, at least at the three-level, by an exchange of massless spin 2 particles, the gravitons. In the static limit the gravitons couple to matter with a strength proportional to the mass. If there exist other bosons, coupling in the same way, then they too contribute to the newtonian limit of gravity. An example is the graviphoton [4], a vector boson present in extended supergravity theories (at least for $N = 2$ and $N = 8$), mediating an interaction which is repulsive between two particles, and attractive between a particle and an antiparticle. In the static limit the graviphoton not only couples to matter with a strength proportional to the mass, like the graviton, but can even lead to antigravity, a cancellation between attractive and repulsive gravitational forces [4].

No significant deviations from general relativity occur at long distances if such bosons are massive, adding Yukawa terms to the static limit of gravity. For masses $m < 10^{-4}$ eV the effective range is $\hbar/mc > 1$ cm, thus in principle observable in the laboratory. Such contributions can be parametrized by a distance dependent gravitational "constant" $G(r)$ [1]. For example, a gravitational potential of the form

$$U(r) = -(G_c M/r)[1 + \alpha \exp(-r/r_0)] \quad (1)$$

results in a gravitational force in the newtonian form

$$F(r) = G(r) Mm/r^2 ,$$

with an effective gravitational constant

$$G(r) = G_c[1 + \alpha(1 + r/r_0) \exp(-r/r_0)] . \quad (2)$$

For a range r_0 of the order of a few meters or kilometers, experimental restrictions on α are very poor. An indirect relation between $G_0 = G(10\,\text{cm}) = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}, the laboratory value, and $G_c = G(10^3\,\text{km})$ follows from the comparison of astrophysical theories and observations. Mikkelson and Newman [1] argued that G_c differs from G_0 by not more than $\approx$40%, using models of the earth and the sun, thus still allowing a substantial deviation between G_c and G_0.

Improvement of the constraints. In the following, arguments will be presented which improve these constraints.

The strength of gravity at distances $\gtrsim 10^3$ km can be measured indirectly by observing stars, since hydrostatic equilibrium plays an essential role in determining their structure. However, stellar models have other uncertainties such as chemical composition and energy generation. In the case of the sun, the helium abundance is not well known, which is reflected in the quoted uncertainty of $\approx$40% in $G_c \approx G_0$ [1]. Moreover, the observed neutrino flux from the sun is far lower than predicted by solar models [5].

These uncertainties can be circumvented to a large extent by examining degenerated stars, such as white dwarfs. Their structure can already be described fairly accurately by assuming the electrons to be completely degenerated and noninteracting [6]. Inclusion of the effect of electrostatic interactions between the electrons and ions, and a few other minor corrections, leads to very accurate models for chemically homogeneous white dwarfs [7,8]. Thus the theoretical understanding of white dwarfs is in much better shape than that of normal stars, where energy generation is of crucial importance to provide the pressure required for supporting hydrostatic equilibrium; or neutron stars, where the equation of state for supernuclear densities is only poorly known.

After completion of the present investigation, the author was informed of several other attempts to use white dwarfs to restrict $G(r)$. Sugimoto [13] used the observational data of Sirius B and o^2 Eri B to argue against $G_c/G_0 = 3/4$, a value proposed by Fujii. Blinnikov [14] used the same white dwarfs to sharpen the restrictions on $G(r)$ to exclude more than $\approx$10% variation. However, he took the radius of Sirius B to be $(0.0078 \pm 0.0002)R_\odot$, an accuracy which seems to be too optimistic [10]. The difference of the present investigation with the two previous ones lies mainly in the use of more recent observational material, together with a detailed motivation and a discussion of the errors involved in the similarity relations [4]. Another very interesting approach is made by Wegner [16][‡1], who has used his observations of the redshift of o^2 Eri B to put very direct limits on G_c/G_0, of an accuracy comparable to those obtained in the present investigation.

In the following we will concentrate on one of the best studied white dwarfs, Sirius B. There are several reasons which make this star an ideal object. The mass is well known from the orbital parameters of the binary system Sirius A and B. During the last few years the effective temperature was determined more accurately, which substantially lowered the uncertainty in the radius. The star is hot enough to avoid surface convection, simplifying the theoretical treatment; and massive enough to guarantee a very thin envelope.

Theoretical models provide us with a relation between the mass M and the radius R of a white dwarf, once the chemical composition is known, to an accuracy of about 0.2% [7], as is confirmed by more recent calculations [8]. In fig. 1 $R(M)$ is given, in solar units, for a pure ^{12}C star (full line) and a pure ^{24}Mg star (dashed). The dotted line indicates the Chandrasekhar approximation (for ^{12}C) of a noninteracting electron gas. The main effect of the correction for Coulomb interaction between ions and electrons is to lower the electron pressure by about 2% (in this mass range), resulting in smaller M and R values for a given central density.

The most accurate observational mass determination of Sirius B [9] is

‡1 Wegner's conclusions are based on observations of the gravitational redshift for the white dwarf o^2 Eri B [17].

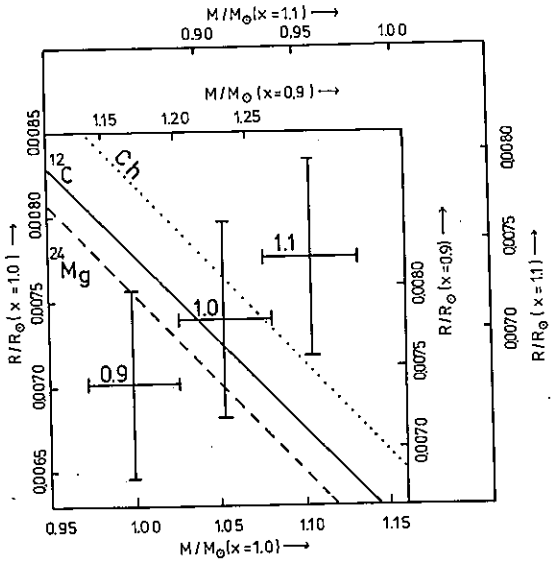

Fig. 1. Observed mass M and radius R of Sirius B, for the three cases $x = G_c/G_0 = 0.9, 1.0, 1.1$. In each case different scales are used along the axes, to keep the theoretical $R(M)$ relations invariant. ^{12}C and ^{24}Mg indicate the compositions of the Hamada–Salpeter models, while Ch indicates the Chandrasekhar model (for ^{12}C).

$$M/M_\odot = 1.053 \pm 0.028 \,.$$

This high accuracy results from the fact that Sirius A and B form a visual binary system, with a period of 50 years, which has been observed extensively over the past century. The radius of Sirius B is estimated [10] to be

$$R/R_\odot = 0.0074 \pm 0.0006 \,,$$

as inferred from its observed luminosity and effective temperature.

Let x be defined by $x = G_c/G_0$, the relative strength of gravity at cosmic distances, compared to laboratory distances. We have to investigate the influence of $x \neq 1$, on the theory as well as the observations, before these can be compared with each other.

The observed mass of Sirius B is derived from a measurement of G_cM, inferred from the binary parameters through Kepler's law. Therefore, the true mass $M(x)$ of Sirius B, as derived from the observations, is proportional to x^{-1}. The ratio $M(x)/M_\odot(x)$, however, remains constant, since the solar mass is derived from observations of $G_cM_\odot$ as well. In the following $M_\odot(1) = 1.989 \times 10^{33}$ g will be denoted by $M_\odot$, for short, as an x-independent reference value. The mass to be inferred from the observations of Sirius B is then

$$M_{\mathrm{obs}}(x) = (1.053 \pm 0.028)x^{-1}M_\odot \,. \qquad (3)$$

Observations of the radius of Sirius B (and of the sun) are not significantly influenced by the value of x, since the radius is determined geometrically from the luminosity and effective temperature (or astrometrically in case of the sun). The only change lies in model atmosphere calculations, where the gravitational acceleration at the surface comes in. However, this x dependence is completely negligible: a change in x of 10% changes R by less than 0.02% [10].

To investigate the effect of $x \neq 1$ on white dwarf models, we will first concentrate on the simpler Chandrasekhar models. They form a one-parameter family of solutions of a differential equation for the degeneracy parameter z, as a function of the distance from the center of the star, where the central degeneracy z_c is the only free parameter. A choice for z_c fixes the entire model, including M and R [6,11].

Since Chandrasekhar's differential equation is written in dimensionless quantities, a value $x \neq 1$ will associate different values of M and R to a particular solution for fixed z_c. Fortunately, the x dependence is very simple, and it turns out that

$$M(z_c;x) = x^{-3/2}M(z_c;1), \quad R(z_c;x) = x^{-1/2}R(z_c;1), \qquad (4)$$

as can be derived from the way the dimensionless quantities scale with G. For a particular value of x, the $M(R;x)$ relation can be obtained from the Chandrasekhar $M(R;1)$ relation, by the use of eq. (4). This is indicated in fig. 1 for the values $x = 0.9$ and $x = 1.1$ by scale-transformations along the axes, in order to leave the dotted line of solutions invariant.

The next step is to evaluate the scaling behaviour of $M(R;x)$ for the more accurate models of Hamada and Salpeter [7]. In principle the x dependence is expected to be more complicated here, but fortunately deviations from eq. (4) turn out to be negligible. By far the most important correction over the Chandrasekhar models results from Coulomb interaction effects, lowering the electron pressure as a function of the density. A variation of 10% in the mass corresponds to a variation in central density of a factor two, changing the Coulomb correction ratio for the pressure by about 0.05% [7,12]. The correction relative to the Chandrasekhar models will then be changed by at most 4%. Therefore the $R(M,x)$ relations for ^{12}C and ^{24}Mg, as given in fig. 1, can be scaled according to eq. (4) to an accuracy of about 0.2% in $R(M,x)$.

For simplicity, three cases are considered, namely $x = 0.9$, 1.0 and 1.1. In fig. 1 the x dependence of the white dwarf models is absorbed into a change of scale along the axes, to keep the model relations invariant. The observations are plotted with respect to the appropriate axis scales, with the mass values corrected according to eq. (3).

In order to use fig. 1 to put limits on x, the chemical composition of Sirius B must be estimated. For white dwarfs with $M \gtrsim 0.75 M_\odot$, only ^{12}C or heavier elements will be present, as indicated by evolutionary arguments [7,8]. It seems likely that in the mass range appropriate for Sirius B ^{12}C will be the dominating element. The pure ^{12}C models, according to fig. 1, restrict x to

$$x = 0.98 \pm 0.07 \,.$$

The sensitivity of this result for variations in chemical composition is not high. Even the extreme case of pure ^{24}Mg (see fig. 1) would lead to $x = 0.96 \pm 0.07$. However, such a heavy composition is far less likely.

Finite temperature effects, which are neglected in the treatment by Hamada and Salpeter [7], were investigated by Lamb and Van Horn [8]. For the observed luminosity of Sirius B, the correction to the radius is an increase of about 0.1%, fully neglegible for the present discussion. Finally, the expected radius $R(M,x)$ will always be slightly higher than calculated in models of pure chemical composition, since the envelope will have admixtures of He and H. This effect is, however, not very important for massive white dwarfs, like Sirius B. Combining all arguments results in

$$x = 0.98 \pm 0.08 \,,$$

where the error is a reasonable estimate of a standard deviation.

In conclusion, theory and observations of Sirius B restrict the value G_c, the gravitational constant at distances 10^3–10^8 km, to agree with the laboratory value G_0 within about 10%. The improvement over previous constraints [1] is due to two factors: the theory of stellar structure is relatively uncomplicated in the case of white dwarfs, and the accuracy of the observations of Sirius B has been improved recently, especially resulting in a more precise value for the radius [10].

These constraints on a possible distance dependence of the gravitational constant, together with more direct observational constraints in the solar system [1] and the most recent direct measurements in the laboratory [15], are summarized in fig. 2. It is clear that every viable (super)gravity theory, in which the static limit of gravity arises from the exchange of other bosons besides the graviton, has to respect these constraints. From fig. 2 it is clear that a coupling strength comparable to that to gravitons, $|\alpha| \simeq 1$, is allowed only for a range $r_0 \lesssim 2$ mm (corresponding to a rest mass of more than 10^{-3} eV). A relative coupling strength of 10% is just barely possible for 1 m $\lesssim r_0 \lesssim$ 10 km.

All these limits apply to the simplest case, where gravity is modified at short distances but in the same ratio for different materials. Much tighter constraints can be set on specific theories which violate the equivalence principle, which is tested to an accuracy of 10^{-12} [18]. An interesting example is the phenomenon of antigravity [4], arising in $N = 2$ and $N = 8$ extended supergravity theories. Here a vector particle, the graviphoton, couples to matter with the same strength as the graviton, at least in the static limit. But the interaction mediated by the graviphoton is repulsive only between particles and particles, or antiparticles and antiparticles, and attractive between particles and antiparticles (in analogy to the coupling of

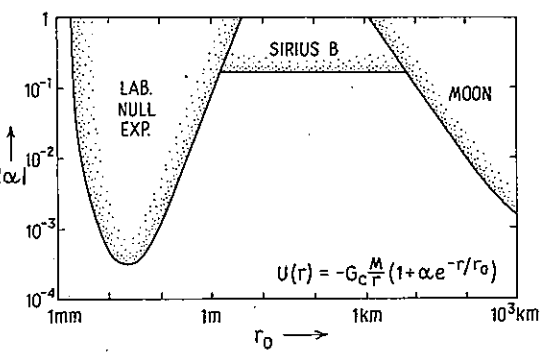

Fig. 2. Limits on the distance dependence of the effective gravitational constant, $G(r)$, parameterized with r_0, the range, and α, the relative coupling strength of an additional component to gravity. The shaded regions are excluded at a 2σ confidence level. The constraints arise from laboratory null measurements [15], surface gravity measurements on the moon [1] and the observations of Sirius B.

photons to charged particles). Scherk [4] assumed the graviphoton to couple to quarks and leptons, which in normal matter are much lighter than the nucleons. Taking the rest mass of the u and d quarks to be roughly 10 MeV he arrived at an upper limit on the range of antigravity of about 2 m (implying a mass of the graviphoton of at least 10^{-6} eV).

Further constraints on this type of antigravity theory follow from fig. 2, since the repulsion between two pieces of matter in bulk has a relative strength

$$\alpha \approx -\left(\frac{3 \times 10\ \mathrm{MeV}}{1\ \mathrm{GeV}}\right)^2 = 10^{-3} .$$

It is clear that a range of roughly 1–10 cm is excluded for $\alpha = 10^{-3}$, but a slightly lower value for the quark masses can easily lower the relative repulsion strength to $\alpha \approx 10^{-4}$, which is not excluded by the laboratory experiments. To improve the restrictions on the range of antigravity, either the null experiments [15] in the laboratory have to be improved, or the equivalence principle [18] has to be tested to even higher accuracy.

In conclusion, every viable (super)gravity theory has to respect the constraints which are summarized in fig. 2. In addition, extra constraints can arise, e.g., from deviations from the equivalence principle, which are dependent on the specific theory. For the antigravity version where the graviphoton couples to quarks and leptons only [4], the allowed range is 10 cm $\lesssim r_0 \lesssim 2$ m and $r_0 \lesssim 1$ cm, the intermediate range being marginally excluded. There are of course many alternative possibilities, e.g., the graviphoton might couple only to subconstituents of quarks and leptons. Although this would change completely the restrictions mentioned above, the effective coupling strength and range would still be subject to the restrictions summarized in fig. 2.

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