NEW DIRECTIONS IN GLOBULAR CLUSTER MODELING

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ABSTRACT. Recent observations show a fascinating and abundant presence of many different types of binaries and binary remnants in globular clusters. What was a perceived dearth of binaries only a few years ago has now turned into a veritable zoo of interesting objects. Besides the, now classic, X-ray binaries, dozens of millisecond pulsars have been detected, scores of blue stragglers, significant numbers of spectroscopic as well as eclipsing variables, and clear evidence for main-sequence broadening caused by the presence of a significant fraction of binaries.

This diverse collection of objects forms a gold mine for dynamicists modeling the evolution of globular clusters. While all these binaries and binary products provide useful diagnostics, some of them are even directly involved in the physical processes of energy generation, driving the core expansion after core collapse. In this sense we are beginning to get a direct look at the central engines which power the later phases in the evolution of globular clusters.

In this paper I first give an overview of globular cluster evolution, during which I discuss two new pieces of research: 1) a general linear stability analysis of the isothermal sphere; and 2) a detailed investigation of the rate of error growth in N-body systems. I then discuss separately six more new directions in globular cluster modeling: 3) the inclusion of a mass spectrum in post-collapse simulations; 4) the addition of primordial binaries in these calculations; 5) new results in binary-binary scattering experiments; 6) the study of hydrodynamic effects during close stellar encounters; 7) approximate migration models for recycled binaries; 8) connections with the dynamics of nuclei of galaxies.

1. Introduction

The last two years have seen an explosion of new activities in the modeling of globular cluster evolution. This has been partly triggered by a wealth of new observational data of various types. These exciting developments appear after a much slower rate of growth of our understanding of star cluster evolution in general, during the last few decades.

In a nutshell, the study of globular cluster evolution began by the late thirties and early forties with the prediction that star clusters would eventually evaporate. Estimates of evaporation rates were worked out further in the fifties. During the nineteen sixties, the mechanism of gravothermal collapse was discovered, and studied in detail in numerical calculations in the seventies. For a long time, the fate of a globular cluster after the initial collapse phase was nearly totally unknown. The eighties finally shed light on this fundamental question, in the form of a number of

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simulations, leading to the discovery of a new physical phenomenon: gravothermal oscillations. So far, these models have been relatively crude, and the main task of the nineteen nineties will be the development of detailed models of post-collapse evolution, detailed enough to be reliably compared with observations.

The organization of the present paper is as follows. In §2 I give an overview of the general area of globular cluster evolution. Our understanding of core collapse is summarized in §2.1, with the addition of some new results concerning the qualitative form of core contraction (§2.1.2), and the rate of error growth in collapse simulations(§2.1.3). The evolution past core collapse is discussed in §2.2. In §3 the first results of post-collapse multi-mass simulations are discused. §4 addresses the implications of primordial binaries for globular cluster evolution. After a qualitative discussion of the main physical effects, the results of detailed simulations are reviewed. Primordial binary observations and their relations with the simulations are briefly mentioned. §5 summarizes the latest developments in binary-binary scattering experiments. §6 reviews some of the recent attempts to begin to understand some of the intricacies of the microscopic processes in star cluster evolution: hydrodynamic interactions during close encounters between stars. §7 addresses the observationally important question of the distribution of binaries through a globular cluster. Recent approximate models are discussed, and compared with observations. A connection between the dynamics of globular clusters and modest galactic nuclei is discussed in §8. §9 sums up.

2. Overview of Globular Cluster Evolution

Globular clusters undergo core collapse on a time scale that is only a few times larger than the half-mass relaxation time. During and after collapse, infinite central density can be avoided only when some form of central energy source turns on. Several energy sources are possible, and they can be divided into three categories. Binaries can increase in binding energy, stars can undergo mass loss, or a black hole may form and subsequently swallow stars. All three processes result in a direct or indirect heating of the cluster.

One of the most interesting new developments is the investigations of the role played by primordial binaries, which can provide 'fossil fuel' to power the central energy source, for a considerable time after core collapse. In the present paper I will only address the theoretical aspects of globular cluster evolution with primordial binaries. For the many exciting new observational results, I refer to an extensive review which will appear separately (Hut *et al.* 1992). A short review of the field of globular cluster evolution was given by Elson *et al.* (1987). A general background for the dynamical evolution of globular clusters can be found in the excellent monograph by Spitzer (1987).

2.1. Core Collapse

The two hundred or so globular star clusters that orbit our galaxy present us with a tantalizing problem. With $\sim 10^5 - 10^6$ member stars each, their dynamics is still too complex for detailed star-by-star simulation on present-day computers. However, much insight has been gained from approximate simulations, based on Fokker-Planck or conducting gas-sphere models, as well as from direct N-body calculations with up to a few thousand particles.

2.1.1. History of Simulations

Half a century ago, Ambartsumian (1938) and Spitzer (1940) independently predicted the inevitable decay of star clusters by evaporation. Two decades later, Antonov (1962) and Lynden-Bell and Wood (1968) showed how a star cluster undergoes internal core collapse, well before its final evaporation, due to the instabilities caused by the negative heat capacity of all self-gravitating systems. Also around this time, Hénon (1961, 1965) constructed the first cluster models exhibiting core collapse.

In the nineteen-seventies, star cluster research received an enormous boost from the unexpected discovery of globular cluster X-ray sources, which were explained as neutron stars that had been tidally captured into binary stars (Fabian *et al.* 1975, Press and Teukolsky 1977). On the theoretical frontier, a variety of numerical simulations began to sketch how star clusters evolve towards core collapse. These simulations were based on the Fokker-Planck approximation for the slow drift of stars in energy and angular momentum space (*cf.* Spitzer 1975). Again a decade ahead of his time, Hénon (1975) was the first to extend a cluster evolution model past core collapse. Soon afterwards, the first observational evidence for a collapsed core, which happened to be in M15, was presented (Newell and O'Neill 1978). It took an additional thirteen years, and the launch of the Hubble Space Telescope, to resolve the core of M15 (Lauer *et al.* 1991).

After a few years of quiet work by theoreticians trying to figure out how to model a cluster after core collapse, the results of a number of different simulations were published around 1984, which were discussed and summarized in papers presented at the I.A.U. symposium 113 (Goodman & Hut 1984). Until 1990, nearly all simulations of star cluster evolution were started with initial conditions in which all stars were single. Stars were allowed to form binaries either dynamically, through near-simultaneous close encounters of three stars leaving two of them bound, or tidally, through energy dissipation in the tidal bulges of one or both of the stars involved in a close encounter. As mentioned above, more information can be found in Elson *et al.* (1987) and Spitzer (1987).

2.1.2. The Qualitative Behavior of the Simulations

The contraction of the inner parts of a star cluster, on a two-body relaxation time scale, is a direct consequence of the negative heat capacity of any self-gravitating system, as follows from the virial theorem. A clear and very physical discussion of the resulting gravothermal instability was given by Lynden-Bell and Wood (1968), who discussed the thermodynamic behavior of an isothermal gas sphere, enclosed in a spherical adiabatic boundary. Although their arguments predicted the overall contraction of the inner parts of a star cluster, they did not address the evolution of the core mass.

Hachisu and Sugimoto (1978) first investigated the form of the run-away solutions, by computing the second order variation in the entropy of the bounded isothermal gas system. A more detailed treatment of the self-similar contraction of the core of a star cluster during the later stages of core collapse was given by Lynden-Bell & Eggleton(1980). They also gave a condition which the heat conduction had to obey in order to give rise to a self-similar contraction of the inner parts of a cluster. They showed how the stellar dynamical process of two-body relaxation can be approximated in the context of a gas sphere, even though the former has a mean-free path much larger than the size of the system, while in the latter this inequality is reversed.

The conditions for core collapse to occur have been further elucidated recently by Makino & Hut (1991). They present a linear stability analysis of the isothermal sphere, for different choices of heat conductivity. Two extreme cases they treat are given by stellar dynamics and by radiative heat transfer. The latter case corresponds to a star starting out with a (nearly) constant temperature distribution throughout its interior. Such a star will start to contract on a thermal time scale, and as a result its core radius, too, shrinks. However, during the contraction the heat conductivity will be more effective in the outer layers than in the inner parts. The reason is that the mean free path length for photons is much larger in the less dense outer layers. Therefore, the innermost layers are slow to respond, and material will rain onto the core from those layers somewhat further out. As a result, the core mass will increase, even though the core radius decreases.

The other extreme case is that of stellar dynamics. Here the heat conductivity has the opposite behavior. At higher densities the frequency of two-body interactions becomes higher, and consequently heat conductivity increases. As a result, the initial contraction will take place more easily and quickly in the core. Indeed, the linear stability analysis of Makino and Hut show how both the radius and the mass of such a system decrease in value. In the non-linear regime, the smaller heat conduction time scale will quickly lead to a core contraction which decouples from the outer layers, leading to the self-similar collapse described by Lynden-Bell and Eggleton.

2.1.3. Limitations of the Simulations: Error Growth

Our understanding of the later stages of core contraction is largely based on Fokker-Planck simulations (Spitzer 1987). This name stems from the fact they are based on a description in terms of statistical diffusion of particle orbits in energy space, described by a Fokker-Planck approximation (Cohn 1979). These methods are quite fast, and have been very fruitful for furthering our understanding of pre-collapse evolution, but they have severe shortcomings when applied to the later phases, as will be discussed below in more detail. Therefore, recent simulations have been increasingly based on direct N-body integration of a system of point masses, under the influence of their mutual gravitational potential.

Even though the N-body problem is easy to formulate, and in fact forms the oldest problem in mathematical physics which could not be solved analytically, it is not easy to simulate numerically either. Not easy, that is, if we insist on getting the correct answer in a strict, mathematical sense of the word. Heggie *et al.* (1988) have shown that a correct calculation which follows an N-body system all the way to core collapse requires a computer calculation with an enormous precision: a word length of roughly N digits! Strictly speaking, this would exclude the simulation of any system with more than 25 or so particles, even when using quadruple precision (128 bit word length).

Fortunately, physicists, and especially astrophysicists, don't always insist on getting correct answers. Good enough answers are good enough for them. The problem is to determine what is good enough. At present, nobody has any formal proof that N-body models give reliable answers, but neither is there any reason to believe that they have any intrinsic bias built in. The bottom line is that there does not seem to be any reason to get worried, although it is interesting to be aware of the articles of faith underlying the simulations. Let me give the main arguments pro and con the reliability of N-body simulations.

The first warning concerning the rate of growth of errors in N-body calculations came from the pioneering investigations by Miller (1964). He considered systems with $N < 32$, and found that the e-folding time scale became shorter with increasing

N, when compared with a crossing time. The dramatic effects of the exponential instability were demonstrated forcefully by Lecar (1968), who coordinated a study, using different computers and algorithms, of the solution generated by one particular choice of initial conditions. In his case, the effect of the instability is intertwined with the effects of different truncation errors and other sources of numerical error. A much later investigation (Heggie *et al.* 1988) showed that the trend found by Miller changes for larger N, to a rate of growth approximately independent of N for $N \gtrsim 30$. They found that in this limit, errors grow with an e-folding time which is nearly one-tenth of a crossing time. More details of their work can be found in Goodman *et al.* (1991). Similar results have been obtained by Kandrup & Smith (1990).

Faced with the impossibility to trace the unique evolution of a star cluster model, starting with a particular set of initial conditions, we can ask whether the (strictly speaking) wrong result of a computer calculation can still be correct in some statistical sense. This question can be asked in two stages. First, we can ask ourselves whether the outcome of an N-body simulation may be the true result of different initial conditions, close to those actually used. This is actually a reasonable guess, since the exponential fanning out of neighboring orbits would imply an convergence upon time reversal of the inspection of a set of forwardly computed orbits [Note that this last point is important: if we would compute the orbits independently backwards in time, we would see an exponential growth towards the past as well, since the equations of motion are invariant under time-reversal].

In practice, errors in conserved quantities such as total energy and angular momentum of the system thwart such a backward shadowing approach, but to within the accuracy of those quantities, it seems likely that a true orbit exists, starting off very near to the actual initial conditions, and leading to the obtained final positions. However, even if such a shadowing theorem could be proved, we would still have no guarantee that the true orbit, although it may be everywhere close to the calculated orbit, would be a generic orbit. In the worst case, if the shadowing orbit would be drawn in a devious way from a pathologic subset of orbits with measure zero, the calculated orbit may have no relevance to the behavior of a typical orbit. However, we have no reason to believe that (computational) nature would be so devious. On the contrary, it is very difficult to imagine how and why such selection effects could take place. While this type of reasoning obviously does not constitute a proof, it will probably be sufficiently reassuring for most physicists.

2.2. Post-Collapse Evolution of Globular Clusters

It was only after the early eighties that simulations became sophisticated enough to be able to penetrate past the near-singular state of core collapse into the asymptotic regime of post-collapse evolution and eventual cluster evaporation (see Goodman & Hut 1985, and references therein). That decade saw a number of exciting theoretical discoveries. Gravothermal oscillations, first predicted on the basis of numerical simulations (Sugimoto & Bettwieser 1983; Bettwieser & Sugimoto 1984), were later also found in semi-analytical studies (Goodman 1987). At about the same time, it was realized that close encounters and physical collisions between stars, previously invoked as possible explanations of cluster X-ray sources and blue stragglers (Fabian *et al.* 1975, Krolik 1983), could also have far-reaching dynamical consequences for the cluster as a whole (Goodman 1984, Ostriker 1985, Lee & Ostriker 1986, Lee 1987ab).

Another important development was the observational discovery of many seeing-limited cores (*cf.* Djorgovski and King 1986), suggestive of remnants of core collapse, and quite well explained in terms of the results of multi-mass cluster simulations, with a judicious choice of stellar mass function (Murphy and Cohn 1988, Murphy *et al.* 1990). Around the same time, evidence was accumulating for the presence of a substantial population of primordial binaries in globular clusters (Latham *et al.* 1985; Pryor *et al.* 1987, 1989; for a review, see Hut *et al.* 1992).

2.2.1. Cluster Expansion

The evolution of a globular cluster after core collapse has only recently been studied intensively, and many aspects of our understanding of it remain uncertain and may change dramatically in the coming years. The *mean* behavior of the cluster after core collapse *is*, however, quite firmly established: the half-mass radius expands according to $r_h(t) \propto t^{2/3}$, where t is the time since core bounce, while the velocity dispersion drops according to $v \propto t^{-1/3}$. This relation may be derived from general principles, without any knowledge of the mechanism of energy generation in the core (*cf.* Hénon 1965, 1975), in a manner analogous to Eddington's (1926) prediction of the mass-luminosity relation for stars, which requires no precise knowledge of the nature of their internal energy generation.

The derivation goes as follows: (1) the half-mass relaxation time t_{hr} in a self-similar solution scales as $t_{hr} \propto t$, the time since core bounce; (2) $t_{hr} \propto N t_{hc}$, where N is the number of stars in the cluster, t_{hc} is the crossing time at the half-mass radius, and we have neglected a factor $\log N$; (3) if we neglect the slow change in mass and particle number due to escape, the virial theorem gives $t_{hc} \propto r_h^{3/2}$; (4) combining these gives $t \propto r_h^{3/2}$ which leads to the results quoted above. In contrast, the rate of expansion of the core *does* depend on the details of the central engine (Cohn 1985; Ostriker 1985). Recently, semi-analytical models for a variety of physical processes and approximations have been developed by Stodółkiewics & Giersz (1990) and Giersz (1990ab).

Thus, regardless of the precise state of the core, and the physical processes going on there, after core collapse, the cluster half-mass radius expands steadily. As it does so, the Galactic tidal field steadily removes the outermost stars, with the result that, eventually, the entire cluster is disrupted. The timescale for this process is longer than the core collapse time, but only by a factor of a few.

What is the character of the central engine in a post-collapse cluster? Several energy sources are possible: one is binding energy extracted from binaries, another is mass loss from the system by stellar evolution. Binaries can be formed by three-body dynamical capture or by two-body tidal capture. Enhanced mass loss can occur when stars collide and merge, forming heavier remnants with a much shorter lifetime under stellar evolution than original cluster stars. Finally, a black hole can form through repeated merging (for a recent review of these three mechanism, see Goodman 1989).

The heating caused by each of these mechanisms takes on quite different forms. Let us first look at binaries. A hard binaries is defined as having a binding energy $>> 1kT$, a measure for the average thermal energy of a single field star. In other words, a hard binary has an orbital velocity clearly exceeding the velocity dispersion of the system (in case of equal mass stars; for unequal masses we have to compare the kinetic energies instead). When a hard binary encounters a single star, it tends to achieve equipartition with the single star, possibly after a temporary capture and/or exchange. In the process it will give off energy to the escaping single star, and harden in the process. In this way, on average the single star comes out of the scattering process with more energy than it went in with. In this way, hard binaries tend to heat their environment.

The second mechanism, mass loss, is a more indirect way of heating a star cluster. Let us take the situation that a cluster loses a fraction ϵ of its mass, either through the escape of one or more stars, or through mass loss through stellar evolution (through a wind, or a Helium flash, or a supernova explosion of a multiple merger product). In all these cases, the mass loss will take place nearly instantaneously with respect to the evolution time scale of the cluster. As a result, the kinetic energy will on average decrease by a factor ϵ (in case of winds or explosive mass loss), or less (in case of a slow diffusion of stars toward unbounded orbits). However, the potential energy of a star cluster is quadratic in its mass, and a decrease by mass of a fraction ϵ will lead to a decrease in potential energy of a fraction 2ϵ. Therefore, the initial virial equilibrium, in which the potential energy was twice the kinetic energy of the cluster, cannot be maintained. After the mass loss, the potential energy will have decreased much more than the kinetic energy. Therefore, effectively, the cluster will have been heated with respect to the new equilibrium situation.

The third mechanism, heating by a black hole, is also an indirect process. The central hole will most likely capture stars which have orbits which are confined predominantly to the central regions of the cluster. Therefore, they carry a relatively small fraction of the kinetic energy of the stellar population. Capturing such stars again tends to increase the relative temperature of the remaining stellar population.

2.2.2. Core Oscillations

In some of the earliest post-core collapse simulations, Sugimoto and Bettwieser (1983; Bettwieser and Sugimoto 1984) found chaotic fluctuations in the size of the core radius. They explained these as a consequence of the gravothermal instability, and therefore introduced the term 'gravothermal oscillations' to describe them. In essence, the underlying physical mechanism can be simply described as follows. For a large number of stars in the system, the inner relaxation time scale is much larger than the half-mass relaxation time scale, which determines the overall rate of expansion. Therefore, the inner regions have the tendency to evolve on a time scale much smaller than the overall expansion time scale. As a result, the inner regions tend to get impatient, and a small fluctuation can trigger a local re-collapse, followed by a local re-expansion. The larger the number of stars, the more the central and outer time scales are decoupled, and the more chaotic the oscillations become. A formal demonstration that the dynamical behavior of these oscillations is characterized by a low-dimensional chaotic attractor is given by Breeden *et al.* (1991; for a summary, see Cohn *et al.* 1991).

The gravothermal character of the core oscillations was confirmed explicitly by Goodman (1987), who performed a linear stability analysis of a new regular self-similar model for post-collapse evolution, and classified the different modes of behavior according to the type of linear instability they exhibit. He found that for a total number of stars $N < N_1$ his self-similarly expanding solution is linearly stable, while for $N_1 < N < N_2$ the solution is overstable, and for $N > N_2$ it is unstable. He estimates $N_1 \approx 7000$ and $N_2 \approx 40,000$. Although the instability for very large N has indeed the expected character of a gravothermal instability, we do not yet understand the nature of the overstability. It is presently somewhat unclear how this behavior is modified by the presence of complicating physical effects, such as a stellar mass spectrum or a substantial binary population.

Like gravothermal collapse, gravothermal oscillations now appear to be a ubiquitous phenomenon, at least in the models which treat the stars and all physical processes as continuous quantities. Inagaki (1986) and McMillan (1986, 1989) have expressed doubts as to whether the oscillations persist in real clusters, where the stars and the physical processes are discrete, and statistical fluctuations may be very large. This issue can probably only be resolved by direct N-body simulations of systems containing $> 10^4$ stars.

2.2.3. Limitations of the Simulations: Time scales and Hydrodynamics

A central problem in globular cluster simulations is the occurrence of widely disparate timescales. Binary stars, with orbital periods of days or less, and separations of fractions of an astronomical unit, play an essential role in the dynamics and the overall energy budget of a cluster. However, the cluster itself evolves on a timescale of 10^9 years, 10^{12} times larger than the orbital period of a tight binary. Comparing the total lifetime of a cluster to the time step needed during periastron passage in an eccentric tight binary, containing two white dwarfs, say, we have a disparity which can easily reach a factor of 10^{18}. It is clear that special measures are needed in order to ensure an accurate treatment of star cluster evolution.

A separate, but equally important, challenge is the extension of our programs to include crude models of physical effects relating to the finite size and internal physics of stars. We wish to incorporate, at least in a rudimentary fashion, the ability to "mix and match" stellar dynamical, stellar evolution, and hydrodynamical codes. One of the most severe requirements here will be that the individual program modules co-exist peacefully during an extended simulation, without the need for intervention by a human supervisor, to guide the codes through the many unforseen bottleneck situations that can reasonably be expected to arise.

For example, allowing arbitrary configurations of binary systems to form, including the possibility of stable (or unstable) mass transfer between any pair of normal, evolved or degenerate stars will surely open a Pandora's box of possible interactions. Add to this the occasional close passage of a third star (or even another binary!) while all this is in progress, and it is clear that we will have to start with quite crude heuristic models, in order to have any hope of success. For example, common-envelope evolution might initially be treated with a rather simple recipe for when, and to what extent, two stars will spiral together, given a particular set of initial conditions. Once the proper overall framework is established, however, more realistic refinements can be added in a relatively straightforward manner.

3. Multi-Mass Models

Until recently, most simulations of the evolution of globular clusters beyond core collapse started off with a population of equal-mass stars. This is not a bad approximation for a first attempt to model an old population of stars, with a turn-off mass around $0.8M_\odot$. However, any detailed comparison with observations requires the inclusion of the effects of the presence of a mass spectrum, especially for the central regions of a cluster where mass segregation plays an important role. The first post-collapse simulations containing a mass spectrum, as well as a large number of other physical effects, were performed in a remarkable investigation by Stodółkiewics (1982, 1985), based on a Monte Carlo Fokker-Planck code. Recently, more detailed multi-mass simulations based on a direct-integration Fokker-Planck approach, have been performed by Murphy *et al.* (1990), Chernoff & Weinberg (1990), and Lee *et al.* (1991).

The simulations by Murphy *et al.* and by Lee *et al.* were based on the Fokker-

Planck approach, in which the central energy generation was modeled on the dynamical formation of binaries in three-body encounters, based on a generalization of the equal-mass expression given by Hut (1985). Murphy *et al.* investigated how gravothermal oscillations are affected by the presence of a mass spectrum. They found that instability occurred for clusters containing more than about $10^5 M_\odot$, with a limiting mass value which had a weak dependence on the steepness of the slope of the initial mass function (note: after publication we realized that we had not used the correct heating rate. As mentioned in more detail by Grabhorn *et al.* 1991, the line in fig. 6 of Murphy *et al.* 1990 should be raised by $30 \sim 40\%$. The predictions for the core radii, however, do not change significantly).

As a consequence, most galactic globular clusters are expected to exhibit instability against core oscillations after core collapse. This has the important observational consequence that the core of a collapsed cluster is likely to be substantially larger than what would be expected from estimates which neglect core oscillations. The reason is that during an oscillation most of the time is spent in the expanded stages, and it is therefore extremely unlikely that we could catch a collapsed cluster in the contracted stage of an oscillation. Instead, we are likely to witness an expanded stage with a core radius typically in the range of $0.05 \sim 0.1$ pc. As a specific example, Murphy *et al.* give a value of 0.07 pc for a cluster with a mass of $4 \times 10^{-4} M_\odot$ and a mass slope of 1.5, close to the Salpeter value of 1.35.

Such relatively large values for the predicted core radii of collapsed clusters are near the edge of observability for typical clusters. It is interesting that the recent HST observations of M15 report a core radius of 0.13 pc, only slightly larger than typical values found by Murphy *et al.* Alternatively, core radii around 0.1 pc can be caused by the presence of primordial binaries, as will be discussed in the next section.

The simulations by Lee *et al.* (1991) include a galactic tidal field, while averaging over gravothermal oscillations. They are thus complementary to the simulations reported by Murphy *et al.* (1990). Lee *et al.* make a number of interesting predictions for the evolution of the mass functions in a globular cluster as a function of radius in the cluster. In addition, they show how during most of the post-collapse evolution the half-mass relaxation time is of order $t_{rh} \sim 0.1t$, with t the age of the cluster. Only during the final evaporation phase does t_{rh} drop significantly, to $t_{rh} < 0.01t$. They compare this with the near absence of galactic globular clusters with $t_{rh} < 10^8$ yr.

Lee *et al.* also studied the effects of the presence of degenerate stars, neutron stars and white dwarfs, many of which are more massive than the average stars and therefore tend to concentrate in the core. This effect became more pronounced in the later stages of their calculations, because the loss of stars over the tidal boundary is more pronounced for the lighter stars which increasingly dominate the outer halo population. An observational effect of a core dominated by dark degenerate remnants would be an increase in the observed core radius, since the light would be less concentrated than the mass.

Chernoff & Weinberg's simulations were also based on a Fokker-Planck approach. They concentrated on the pre-collapse phase, and made a detailed study of the effects of mass loss by stellar evolution as well as by escaping stars, through the dynamical response of the tidal radius of a globular cluster. In addition, they presented detailed information about observational aspects of their models, in terms of colors, surface brightness, mass-to-light ratios, and mass functions. They pointed out that these realistic detailed modeling studies complement earlier, more schematic studies of the galactic family of globular clusters as a whole (*cf.* Chernoff *et al.* 1986, Chernoff and Shapiro 1987, Aguilar *et al.* 1988, Chernoff & Djorgovski 1989).

Intriguing as the above Fokker-Planck results are, it is important to realize how many effects have not yet been modeled accurately. One limitation lies in their statistical nature, which is of questionable validity after core collapse, when individual binaries can release large amounts of energy. The resulting recoil will introduce large changes in the orbital motions of single stars and centers-of-mass of binaries, in direct contradiction of the underlying Fokker-Planck assumptions. A second limitation for realistic applications lies in the relatively small number of physical parameters that can be modeled statistically. For example, providing statistical bins simultaneously for a collection of mass, energy and angular momentum choices will result in many bins containing only a fraction of one star, obviously jeopardizing the statistical assumptions of a Fokker-Planck code. These problems can be overcome by direct N-body calculations, but so far the necessary computer power, in the range of Teraflop-days (Hut *et al.* 1988).

4. Primordial Binaries

Globular clusters contain relatively fewer binaries than the galactic disk, and until only a few years ago it was not clear whether any stars had been formed in binaries at all, during the birth of globular clusters. Recently, however, observations have indicated that the number of primordial binaries is large enough to have an important influence on the dynamical evolution of globulars. This has been the motivation for several detailed simulations modeling such an evolution in the presence of primordial binaries.

4.1. Observations

Only a decade ago, there was no positive evidence that globular clusters contained significant number of binaries other than those made dynamically through two-body and three-body capture. This was rather remarkable, since in the solar neighborhood binarism is the rule rather than the exception. The first systematic search for radial velocity variables (Gunn & Griffin 1979) did not result in any positive detection, and for a number of years it was widely believed that globular clusters were severely binary-poor.

A few years later, two dwarf novae were detected in a globular cluster (Margon *et al.* 1981; Margon & Downes 1983). However, these cataclysmic binaries contain white dwarfs and may well have been formed through tidal capture in a way similar to the formation of their neutron star containing counterparts, the low-mass X-ray binaries. Thus, still no observational handle was available on the presence or absence of primordial binaries in globular clusters. This situation began to change soon after the first discovery of a red giant star showing a variable radial velocity with a strong indication for a binary orbit (Latham *et al.* 1985). A few years later, the first non-zero estimate of a primordial binary population in globular clusters was given by Pryor *et al.* (1989): they estimated that $\sim 10\%$ of all the stars in the then surveyed clusters were the primary of a binary.

Meanwhile, another exciting development was the discovery of a large number of millisecond pulsars, starting with the discovery of 3 ms pulsar in M 28 (Lyne *et al.* 1987; for a recent update, see his contribution to the present proceedings). Again, these pulsars do not provide a direct handle on the abundance of primordial binaries, since they clearly have been recycled. Indirectly, however, they do suggest that primordial binaries do play an important role in their dynamics (*cf.* the contribution by Phinney in the present proceedings, and Hut *et al.* 1991a).

Finally, the last few years have witnessed an explosion of new observational results. Besides X-ray binaries, cataclysmic variables, millisecond pulsars and radial velocity variables, we now have firm evidence for many newly discovered eclipsing variables, as well as for a significant broadening of the main sequence in the color-magnitude diagram of several globular clusters, indicating the presence of significant numbers of primordial binaries. An extensive review of all these new observational developments, together with references to the original literature, is given by Hut *et al.* (1992).

4.2. N-Body Simulations

Until recently, most N-body calculations started off from a cluster model which contained only single stars. At the late stages of core collapse, one or more binaries were formed dynamically in a simultaneous close encounter of three stars. The energy released by these stars reversed the collapse to a slow expansion of the whole cluster. For information about single-star runs, see the review by Aarseth (1985), and earlier references therein.

The earliest N-body simulations which began with a significant number of primordial binaries, are those by Aarseth (1980) and Giannone & Molteni (1985). Aarseth started his 250-body calculations with eight primordial binaries. Giannone and Molteni used a similar number of particles, while increasing the number of primordial binaries to sixty. A few years later, Leonard and Duncan (1988, 1990) published a number of runs aimed at modeling young galactic clusters rather than old globular clusters. Therefore, they contained a large fraction of stars in binaries (2/3 by mass). Because of the large computational cost implied, the total number of stars they used was relatively modest ($N = 45$), but appropriate for their main interest, which was a study of the properties of the escapers.

The most extensive N-body calculations containing primordial binaries have been published recently by McMillan *et al.* (1990, 1991). They modeled the evolution of equal-mass star clusters containing a mass fraction of approximately 20% binaries, containing more than $N = 1,000$ stars. Some of the runs extended to very late times, past the point were all the primordial binary 'fuel' was burned up, at which time the cluster had to manufacture new binaries in three-body encounters. For comparison, they also ran simulations with the same binary percentage but a smaller overall number of stars, as well as simulations without primordial binaries.

McMillan *et al.* found the following global response of a cluster to the presence of primordial binaries. The pre-core-collapse evolution was driven by mass segregation between the equal-mass single stars and the binaries, which were twice as heavy. After core collapse, the cluster showed, on average, a smooth reexpansion driven by a steady rate of burning (*i. e.* hardening) of primordial binaries. With so much primordial fuel present, the post-collapse cluster core was significantly larger than was the case in comparison runs without primordial binaries. On a time scale more than an order of magnitude larger than the original collapse time, most binaries were destroyed as a consequence of binary-binary collisions. The surviving binaries showed a gradual hardening. Typically, core collapse occurred, in a 1000 particle system with 100 mildly hard binaries, after $50t_{cr}$ (half-mass crossing times). By this time about one third of the binaries had been destroyed, but without giving off much energy. By $t = 150t_{cr}$, another one third of the binaries had been destroyed, this time accompanied by a significant production of energy. Of the remaining one third of the original binaries, half again disappeared, through destruction and escape, by $t = 500t_{cr}$ (the half-mass relaxation time is $t_{hr} \approx 10t_{cr}$).

An interesting aspect of the simulations of McMillan *et al.* was the smooth statistical nature of the process of energy production by the binaries. Previous runs had always shown strong perturbations, of order unity, of the core due to individual strong scattering events involving a single hard binary in the core. These perturbations will become smaller with larger N values, but only very slowly, since N_c, the number of core particles, shows only a weak dependence on the total number of particles: $N_c \propto N^{1/3}$ (Goodman, 1984). The reason is that the three-body mechanism for producing new binaries is highly density-sensitive. Even though the required relative energy production rate per half-mass crossing time drops linearly with N, the core cannot grow much without falling behind its obligatory energy generation rate.

This situation is very different with the presence of primordial binaries, where binary–single-star encounters and binary-binary encounters can generate heat, without any need for encounters of three independent bodies. As a result, even a modest amount of primordial binaries (a few percent will do) will give rise to a core population growing linearly with cluster mass: $N_c \propto N$ (Goodman & Hut 1989). As a result of this larger core size, fluctuations due to strong interactions involving individual binaries do not cause such large perturbations for the core as a whole. This is the reason that the simulations by McMillan *et al.* are the first ones to enter the regime in which the core shows a smooth evolution, with fluctuations significantly less than order unity. Consequently, many theoretical notions for large-N clusters can now be tested directly, rather than through Fokker-Planck simulations.

An example of such detailed analysis is the ratio of the core radius to the half-mass radius, about 0.1 immediately after core collapse in the simulations by McMillan *et al.* , dropping to about 0.05 by the time most of the binaries were destroyed. The core mass similarly dropped by a factor of two during this time. The mass fraction of binaries in the core showed a steady, near-linear decrease from 0.5 to 0.1 between $\tau \sim 20$ and $\tau \sim 250$, where the time increment $d\tau$ is measured in units of the instantaneous half-mass crossing time.

In addition, McMillan *et al.* obtained a wealth of microscopic information concerning individual interactions between single stars, binaries, triples and occasional more complex multiple star systems. They presented a study of binary interactions in the context of a real cluster environment, and compared their results with earlier "laboratory" simulations, in which similar interactions were studied in isolation (see §4.4). In addition, they studied the formation and evolution of hierarchical triple systems, and the generation of energy within short-lived overdense "clumps" of stars, rather than via binary interactions in an otherwise smooth background.

The main shortcoming of N-body calculations so far has been the relatively small N values, some two orders lower than typical globular clusters. Fortunately, computer power is growing rapidly, and as a consequence, the nineties may provide us with the first opportunity to model a globular cluster on a star-by-star basis. The hardware requirements of computing speeds in the Teraflop domain (Hut *et al.* 1988) may begin to become accessible in the coming years. An interesting approach to reaching such high speed has been pioneered in the GRAPE project (from "GRAvity PipE") at Tokyo University (Sugimoto *et al.* 1990), through the development of special-purpose hardware in the form of parallel Newtonian-force-accelerators, in analogy to the idea of using floating-point accelerators to speed up workstations. For more information about the GRAPE hardware design, see Ito *et al.* (1990, 1991). For software aspects, and choice of algorithms, see Makino *et al.* (1990) and Makino (1991).

4.3. Fokker-Planck Simulations

As long as N-body simulations are limited to at most a few thousand particles, Fokker-Planck simulations provide the most accurate way to model realistic N values for globular clusters, in the range $N = 10^5 \sim 10^6$. Earlier Fokker-Planck simulations containing primordial binaries were reported by Spitzer & Mathieu (1980). They did not find a reversal of core collapse, probably because their calculations did not extend far enough (*cf.* Spitzer 1987). Hills (1975) described an analytical model for the evolution of a cluster core containing initial binaries, but his choice of scaling for core quantities, $R_{core} \propto N_{core}^2$, turned out to be unphysical (R_{core} scales roughly proportional to N_{core} for an almost isothermal collapse). Summaries of these investigations, as well as of the early N-body explorations using primordial binaries mentioned above, can be found in the introductory section of McMillan *et al.* (1990) and Gao *et al.* (1991).

The first, and so far only, Fokker-Planck simulations with primordial binaries which extended past core collapse are the ones reported by Gao *et al.* (1991). Previous Fokker-Planck simulations (see Murphy *et al.* 1990 and references therein) had modeled energy generation only in a rather indirect way. The trick had been to estimate the net rate of energy release from binary–single-star scattering through an estimate of the bottleneck factor: the formation of new binaries in simultaneous encounters of three single stars. The latter rate had been estimated by Goodman and Hut (Hut 1985).

The problem with the presence of primordial binaries is that this trick does not work any more, since no such bottleneck is present anymore. Instead, a more accurate treatment would suggest the introduction of a two-dimensional distribution function for the binaries, with the binding energy and the energy of the center-of-mass motion as the two independent variables. However, to avoid the expense and complexity of a two-dimensional computational grid, Gao *et al.* took a short-cut by introducing a factorization of the distribution function in terms of two one-dimensional distribution functions, one for the internal and one for the external energy of the binaries. They justified this approach by noting that the two-body relaxation time is typically much shorter than the time between successive strong interactions of binaries.

They also noted the shortcomings of such an approach, in that it could not treat the ejection of binaries on elongated orbits far into the halo. As we will see in §7, such orbits do play an important role, in the simulations themselves, as well as in providing important observational handles on the evolution of globular clusters. But, as they mentioned in their discussion, an accurate description of these 'halo parking orbits' would require a three-dimensional distribution function, including not only internal and external energy but also the external angular momentum of the center-of-mass orbit. Clearly, such a treatment is not practical within the Fokker-Planck approach – not in the least because a typical cell in a three-dimensional computational grid would contain much less than one binary at any given time, making the statistical interpretation completely unrealistic. In this sense, it is clear that the simulations by Gao *et al.* , ingenious and interesting as they are, have pushed the range of applicability of Fokker-Planck treatments to the edge of what is compatible with the underlying assumptions.

One advantage of the Fokker-Planck approach of Gao *et al.* is that the large effective-N values allowed them to study the occurrence of gravothermal oscillations, which could not be studied in the relatively small-N simulations by direct integration of McMillan *et al.* They found that the primordial binary population was able to suppress these oscillations during a period equal to several initial core collapse times.

After this time, with most primordial binaries having been burned up, the core would shrink enough to allow the gravothermal oscillation instability to occur.

5. Binary-Binary Scattering

In this section I summarize the results of some of the binary-binary scattering experiments which I performed a couple years ago, and which have not been published earlier. Originally, this summary was scheduled to appear in the proceedings of the workshop on Self-Gravitating Systems in Astrophysics and Nonequilibrium Processes in Physics, which was held in Kyoto in 1989 (McMillan *et al.* 1990, 1991 refer to this reference). However, since I understand that no proceedings will be published for this meeting, I include my intended contribution here in the form of the present section.

Gravitational scattering experiments have been performed and analyzed with a new code which is capable of treating any type of self-gravitating target or projectile composed of point masses, with no restrictions on the number of particles or the hierarchical structure of their internal orbits. This computer code forms an important ingredient in a computational laboratory for stellar dynamics. The most complex part of the code is the module which recognizes the emerging of stable final states signaling the end of a scattering experiment. The analysis needed for the recognition is based on a mixture of theoretical, heuristic and empirical reasoning.

A first application of the code is discussed, in the form of a study of binary-binary encounters. Such encounters will be important in the dynamics of globular clusters if primordial binaries contain a few percent of the stars of a typical cluster, as has been suggested by recent observations. In this case, mass segregation will cause the heavier binaries to concentrate in the core and many binary encounters will involve other binaries rather than single stars. Some binary-binary scattering cross sections are presented which have been recently obtained. These can be used to estimate the relative importance of different types of scattering processes.

Another application of the new code will be a study of the formation and destruction of hierarchical triples which cannot be formed in three-body scattering events, but only in those involving four or more bodies. Additional subjects for study are binary binaries (two binaries in a stable orbit around each other), and even more complex systems, which are likely to be produced in a cluster core which is saturated with primordial binaries.

Finally, the software which handles the recognition and classification of hierarchical structure will have several applications in the context of larger N-body codes. It will provide a general way to handle the regularization and other special treatments which are needed to retain accuracy in the integration of the particle orbits in dense subsystems. It also will enable detailed analysis and diagnostics of the precise nature of the multiple encounters which fuel the post-collapse expansion in models of globular clusters.

5.1. Introduction

Scattering experiments play an important role in many areas of microscopic physics. In most systems the interactions of individual constituents display a bewildering variety of behavior when investigated in detail. In general, such information is not relevant for a global, thermodynamic description of the system. What we are mainly interested in can often be expressed by a few numbers, such as the heat capacity and the heat conductivity. A convenient way to arrive at such final bulk numbers is a three-step approach: (1) one starts by measuring a few relevant numbers at the end of each individual scattering experiment, where these numbers are the values of some quantities which are judiciously chosen so as to shed maximum light on the property of the system one is after; (2) the next step involves a statistical averaging (ensuring a proper weighting procedure) of all individual results in similar runs in order to express all these results in the form of a few cross sections and reaction rates; (3) the final step is to start with the statistical nature of the individual constituents, as characterized by the cross sections and reaction rates, and to average their behavior over the system as a whole, by using the known statistical distributions of properties of the constituents.

An astrophysical example of an application of microscopic scattering experiments is given by the low energy nuclear physics experiments which are performed to determine the cross sections and reaction rates which are used as input in stellar evolution calculations. In this case, steps 1) and 2) of the classification given above are performed in the laboratory while step 3) is realized on a computer in a stellar evolution code.

In other areas of astrophysics, scattering experiments themselves can be of macroscopic scales. Such experiments are purely computational by necessity, whenever the individual experiments are impractical. The subject or gravitational scattering falls in this category, since encounters between single stars and double stars, or for that matter between whole galaxies, are not easy to orchestrate in the real world. In the remainder of this paper, we will first give an overview of the software implementation of steps 1)-3), followed by a discussion of a specific application, namely globular clusters with primordial binaries.

5.2. Gravitational Scattering

Gravitational scattering studies the interaction and final states of encounters between two or more objects, each of which can be a single star, a double star or a hierarchical multiple such as a triple or a binary binary. Although the study of gravitational bound states lies at the very beginning of mathematical physics, gravitational scattering is a very young field. Only for the simplest case, binary - single star encounters in the limit of all three stars having equal mass, a complete theoretical and experimental treatment is available. For some other cases, such as more general binary - single star scattering and binary - binary scattering, some quantitative results have appeared, but even here detailed knowledge of cross sections and reaction rates is still lacking. For all other cases, not only is quantitative knowledge nonexistent, even a qualitative description, notation and classification is lacking. Fig. 1 summarizes the basic processes which can occur in gravitational few-body scattering.

A specific example of an application of gravitational scattering in astrophysics is based on the gravitational three-body problem. After measuring the outcome of orbit calculations for individual encounters between single stars and binary stars, it is possible to produce statistical expressions, in a given environment, for the average rate of increase of binary binding energy and the resultant rate of heating of the stellar environment. In the three-step classification given above, step 1) is relatively simple when we limit ourselves to gravitationally interacting point masses. Step 3) consists of writing a computer code which can follow the evolution of a star cluster in a statistical way, for example in the Fokker-Planck approximation. After core collapse, a star cluster can begin to re-expand when binaries are formed dynamically and begin to give off excess heat to the environment when their binding energy increases on average. A major complication, however, arises at step 2): how do we insure a proper sampling of initial conditions, as well as a proper statistical interpretation of the final results of the individual experiments?

Some of the questions involved in step 2) have been addressed by Hut and Bahcall (1983), in an extensive project of numerical orbit integrations to determine cross sections of scattering processes between single stars and binaries. In this paper an outline and background of the three-body scattering project can be found, together with technical details concerning the numerical scattering experiments of single star incident on binary stars. Also in this paper are presented the main results of the first 1.7 million scattering experiments, for equal masses and both soft and intermediate binaries. More detailed results can be found in Hut (1984), which includes an atlas of differential cross sections for a wide variety of different processes. For a modern mathematical discussion of the general three-body problem, see Marchal (1990). See also the reviews by Valtonen (1988) and Anosova (1990).

5.3. N–Body Scattering for $N > 3$

In general N-body scattering experiments, progress has been much more modest, compared to binary-single-star scattering. Some binary-binary scattering experiments have been reported in the literature, notably by Mikkola (1983ab, 1984ab), Hoffer (1983, 1986) and recently Leonard and Duncan (1990). For a more complete list of references, *cf.* the review articles by Hut (1985); Elson *et al.* (1987); Hut *et al* (1988); Valtonen and Mikkola (1990). However, the results of the published scattering experiments were not presented in such a way as to allow computation of scattering cross sections and reaction rates, as is customarily done in other branches of physics. Fig. 2 presents the first result along these lines, and will be discussed below.

One reason that N-body scattering experiments for $N > 4$ have not been reported earlier is the enormous complexity of the internal dynamics of the scattering process itself, as well as the large number of possible final states. Writing software intelligent enough to know when to terminate such a scattering process is a nontrivial exercise in qualitative reasoning. When an interacting group of particles breaks up in separate clumps, under which conditions is each clump stable against further disintegration? And under which conditions can we guarantee that none of the clumps will undergo a further encounter with any of the other clumps? Several minimal criteria will come to mind, such as the requirement that the system of clumps is unbound, and that the same holds for each pair of clumps. In addition, it seems safest to require that all clumps move radially outwards, and that each clump moves away from each other clump. However, further thought (and experimentation!) shows that some of these requirements are too rigid, and also that there still are some loopholes left open, allowing further interactions between clumps to take place.

To give an example of a too rigid restriction, consider clumps moving away in opposite directions at great speed, and a third clump moving away slowly from the previous interaction area, but then reversing its direction under the influence of the slight attraction of the receding clumps. In this case, the inter-clump interaction is clearly over, but it can take a very long time before the third clump has crossed the interaction region, and is finally moving out again.

An example of a loophole is the following. Consider two clumps moving away in nearly parallel orbits, slightly diverging so as to give them a relative motion which is receding. Assume them to be unbound, which guarantees that, by themselves, they will forever continue receding. However, there is still the possibility of a third

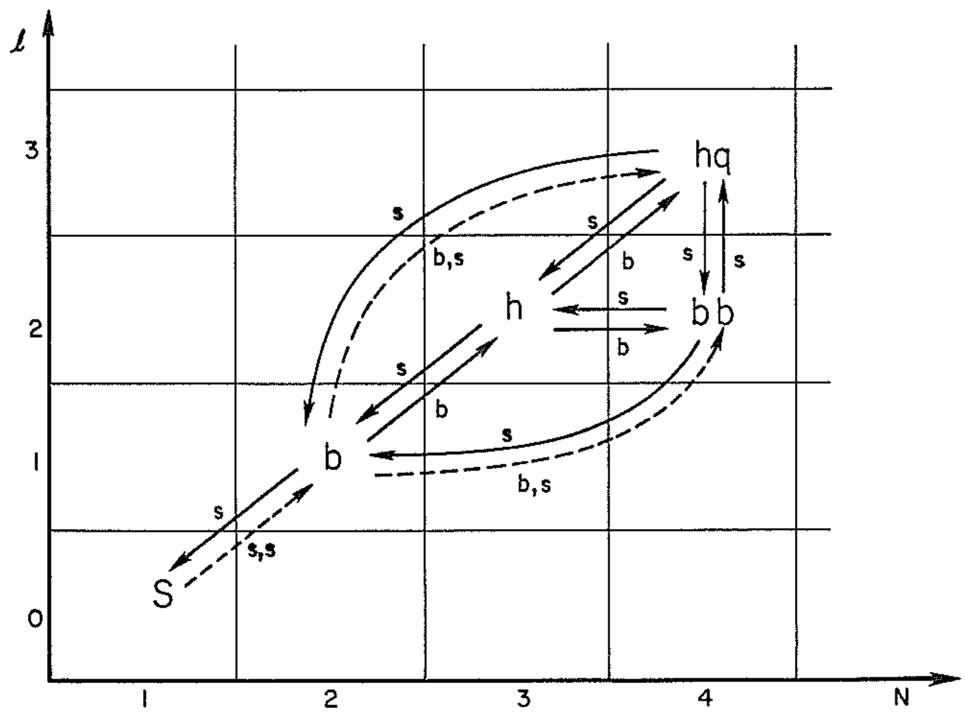

Fig.1 A chart of gravitational isotopes: stably bound systems of N point masses, in a hierarchy of l levels deep. s denotes a single star; b a binary, h a hierarchical triple, *i.e.* a single star in orbit around a binary; bb a binary binary, *i.e.* a second-order binary each of its members being a binary itself; hq a hierarchical quadruple, *i.e.* a second order hierarchical system, composed of a single star orbiting a simple hierarchical system of type h. Encounters between two objects can result in transitions indicated by the full lines; encounters which require the simultaneous interaction of three objects are indicated by dashed lines. In all transitions starting at a particular object, the additional objects involved in the encounter are listed alongside the arrow which describes the transition. Note the existence of a bottle neck in the transitions between single stars and binaries. This process is a "forbidden transition" from the point of view of the dominant two-object encounters, and requires the near-simultaneous presence of three unrelated objects (single stars in this case).

clump (or combination of several other clumps) exerting just enough force to focus the two escaping clump orbits onto each other, even though these residual forces are guaranteed not to hinder escape of the two clumps from the system as a whole. In this rare case, subsequent interaction between the clumps cannot be excluded.

These examples are mentioned here just to give an impression of the complexity of orchestrating individual scattering experiments in an automatized way. In addition, there is the problem of orchestrating whole series of scattering runs. These problems are very similar to the question of how to organize laboratory experiments in general.

5.4. A Computational Laboratory

Gravitational scattering experiments can be performed on a computer in a variety of ways. The simplest approach is to write a computer code which is restricted to solving the equations of motion and propagating the particles, while giving periodic output of the intermediate results. A human controller can then study the successive output results and determine at which point the calculation has run its course and should be terminated. Let us first look at scattering experiments involving whole galaxy models, rather than only a few particles such as in three-body scattering experiments.

In the galactic case, the criterion for halting the calculations can be relatively simple. One possibility would be simply to take the separation of two galaxies beyond a certain distance, in the case of a high-speed encounter between two galaxies where it is *a priori* known that the two galaxies will not stick together. A more complicated criterion is necessary in the case that the encounter speed is low enough to allow the possibility (but not the certainty!) of a merging of the two galaxies. Not only is a decision needed as to which of the two outcomes has been established (merging or escape), also there is a lot of freedom in determining a halting criterion in the case of a merger. It is sensible to continue the run as long as the merger remnant still shows clear signs of evolution, of not yet having settled down in its new form. However, the translation of this qualitative remark into hard objective criteria is not so simple, and certainly not unique.

This brings us to a second type of approach to performing gravitational scattering experiments on a computer. We can extend the task of the computer, by writing a second code which controls the action of the code used for getting the computational crunch work done. Again, depending on the subtleties involved in deciding when to end a calculation, this second-level code can be short and simple, or it can be longer and more complicated than the first-level code, for example if something like pattern recognition is involved in classifying the status of the system at a given time (*cf.* the discussion in the previous section).

The task of the human researcher is obviously greatly simplified with the halting decisions for each experiment being taken care of by the computer. What remains to be done by hand, though, is to choose a set of initial conditions for each experiment which will be carried out. This third-level type of activity can be automated as well, as will be discussed briefly below.

5.5. A Laboratory Assistant

The central problem in automatizing the setup of experiments is formed by the much more extended amount of knowledge which needs to be built into the computer program. On the first level, the program should have a working understanding only of the equations of motion. On the second level, not only the mechanics but also the aim of each experiment should be coded into the program. For example, the question of what exactly we want to measure to what (quantitative or qualitative) accuracy at the end of a calculation can cause the structure of the second-level program to be vastly different for different types of experiments, even when the first level program remains completely unchanged.

At the third level, we try to automatize not only the halting but also the starting-up and the choice of start-up parameters of each experiment. This requires a much wider type of knowledge than that used on the second level. In laboratory terms, a machine to perform the experiments is often modified in such a way that it can also signal the end of an experiment, and in addition that it can do some of the early data reduction and interpretation as well. However, the choice of a complicated suite of experiments is often left to a human laboratory assistant, rather than a computer (although this situation is slowly changing).

One important distinction to make at this level is that between pilot studies and production runs. When we start a new series of experiments, it may well be that we do not know the precise part of parameter space which we will want to search systematically. Instead, we may want to do a few "shots in the dark", to get a preliminary feeling for the relationship between the choice of initial conditions and the type of outcome of the experiments. Once we have a clear enough picture, we can start to carry out production runs, in which we systematically explore the parameter range which seemed appropriate as a result of the pilot studies. In general it is a good idea to use these productions runs also to keep checking the appropriateness of the calculations, as was done first in the pilot studies.

Of these two types of third-level activities, perhaps the pilot studies are the most complex from a computer science point of view (as well as from a coding point of view). The reason is that at this point most is demanded in terms of imagination and creativity, because the outcome of experiments is completely new and different parameter regimes are often unknown until the pilot experiments are actually performed. What is needed goes beyond the more familiar rule-based expert systems, in that some of the rules of the game have to be determined empirically, while performing pilot experiments.

5.6. Applications: Primordial Binaries

The software tools described above have been under development over the last several years, and have recently reached a stage in which they can be applied to real problems. Specifically, the computational scattering laboratory for general N-body scattering saw its "first light" recently, with the successful completion of initial runs of binary-binary and binary-triple scattering. Here we will give some preliminary results of the binary-binary scattering experiments.

In the dense cores of globular clusters gravitational encounters occur in a bewildering variety when mass segregation has produced a stellar population which is locally dominated by binaries. Fig. 2 presents some preliminary results which indicate the relative importance of different type of scattering processes. In Fig. 2b, I confirm in quantitative detail the result reported first by Mikkola (1983ab), that binary-binary encounters produce significant numbers of hierarchical triples. In a binary-saturated cluster core, these triples themselves undergo subsequent encounters, leading to an enormous variety of formation scenario's for observable objects such as millisecond pulsars and low-mass X-ray binaries, some of which may well have a third companion star.

Fig. 2 Preliminary results for scattering cross sections in binary-binary scattering. All masses are equal and the two binaries have identical semimajor axes a. Their eccentricities differ, and are drawn independently from a thermal distribution, flat in e^2. The cross sections are normalized by multiplying them with the square of the relative velocity between the two binaries, as expressed in the system of units used by Hut and Bahcall (1983; $v_{cr} = 1$ in equal-mass binary-binary scattering).

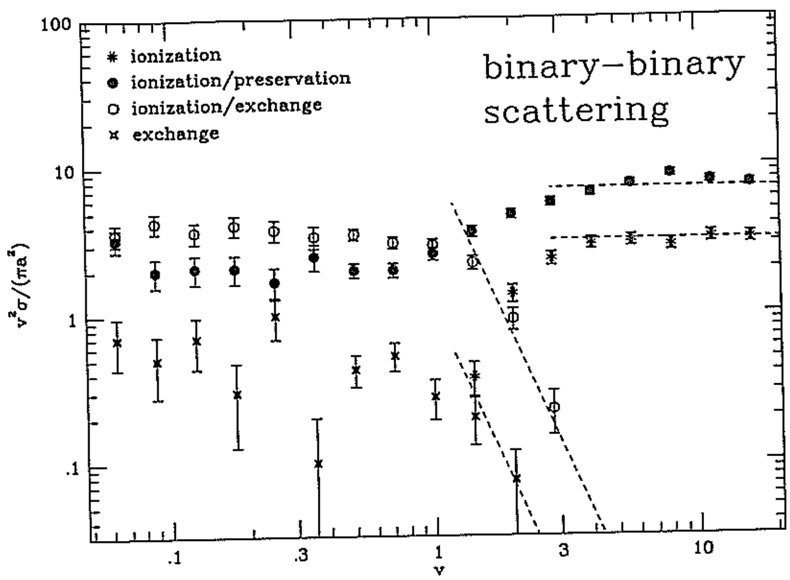

Fig. 2a. Four types of scattering cross sections are presented, as a function of relative velocity of binaries: 1) ionization, in which both binaries are broken up into single stars; 2) ionization/preservation, in which one of the binaries is broken up while the other retains its original composition; 3) ionization/exchange, in which two single stars emerge and one binary, formed by members drawn from both original binaries; 4) exchange, in which two new binaries emerge, each containing members from both original binaries. The dashed lines indicate theoretical scaling laws. These will be refined and presented in a detailed paper, to be submitted to *Astrophys. J.*

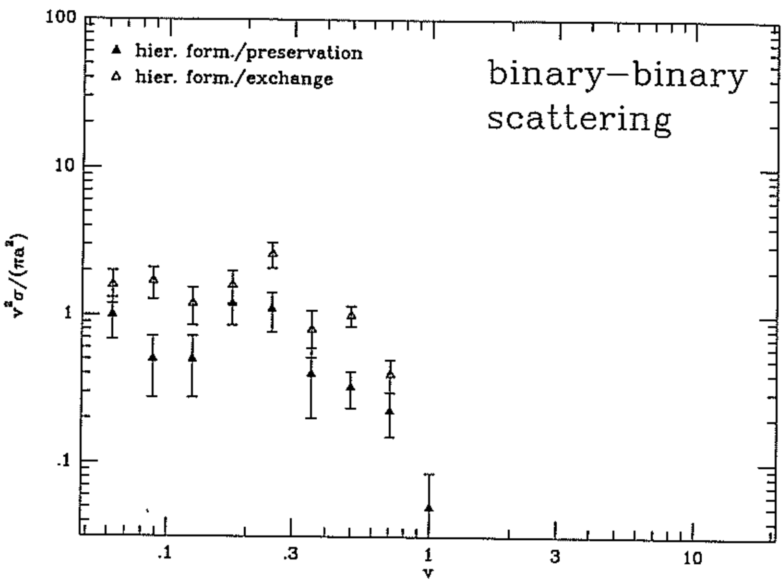

Fig. 2b. The remaining two types of scattering cross sections indicate the occurrence of formation of hierarchical triples. Preservation denotes the survival of one of the two binaries in the form of the inner binary of the triple. The alternative in which the inner binary has received members from each original binary is called exchange. Note the frequent occurrence of hierarchical triple formation, compared to the other processes depicted in fig. 2a, in the left-hand part of the figure, which describes collisions between hard binaries.

5.7. Future developments

So far our discussion has been based on interactions between point particles. In many astrophysical applications, however, we cannot neglect the physical size of the stars, because the probability of collisions is appreciable. In addition, an immediate consequence of introducing more realistic star models is the possibility of energy dissipation, even without physical collisions. For example, in a typical globular cluster theoretical models predict that hundreds, or perhaps even thousands, of binaries are formed by tidal capture even before core collapse, and a comparable number of stars will actually collide and merge. Such processes are not only interesting in themselves, but they will also have an important influence on the overall dynamics of the whole star cluster. Therefore, if we are to make a realistic globular cluster evolution model, it will be essential to take such processes into account.

A detailed realistic description of a collision between two stars would involve a three-dimensional hydrodynamic code which includes an accurate treatment of shocks, radiation transport, nuclear reactions, convection, etc. Even a collision between two stars will in general result in a merger remnant with a mass less than the combined mass of the progenitors, even if we wait sufficiently long for the gas clouds in bound orbits to fall back on the merged stars. The remaining matter will leave the interaction area, and may leave the star cluster under consideration (in case of a globular cluster), or may be reprocessed in some form (as could occur in a galactic nucleus). In a globular cluster simulation, we could decide to follow only what happens in a limited volume of space time centered on a scattering event, while simply neglecting the gas clouds and radiation emitted in the process and concentrating on the remaining merger remnants.

Until recently, available computer power was too limited to attempt three-dimensional hydrodynamical calculations of stellar interactions. However, the recent development of particle-based schemes such as Smooth Particle Hydrodynamics (*cf.* Monaghan 1985, Hernquist and Katz 1988) together with the increase in speed (as well as availability!) of supercomputers, may make a systematic study of fully 3-D stellar encounters possible.

6. Microscopic Physics

Most globular cluster model calculations so far have used the point-mass approximation to represent individual stars. At first sight, this is reasonable for a typical globular cluster star, since the chance for a physical collision or close encounter with another star is small. However, in the central regions finite-star-size effects become important.

6.1. Stellar Finite-Size Effects

Let us make a simple estimate of the importance of non-point-mass effects. For comparison, take a star at the half-mass radius of a globular cluster, with a velocity dispersion of 10 km/s. With interstellar separations of $\sim$ 0.2 pc, it takes a star 20,000 years to cross an interstellar distance. But the radius of the target area is small, about 0.3 A.U. (the geometric mean between the sum of the diameters of the stars and the ninety degree turnaround distance, because of gravitational focusing). With an interstellar distance of 40,000 A.U., the chance to hit the next star is less than 10^{-10}. The chance to have an encounter after 10^{10} yr is thus less than 0.005 %.

These arguments indicate that the early stages of globular cluster evolution, up to core collapse, can be well approximated by point-mass dynamics. The main mechanisms, evaporation and the onset of the gravothermal instability, are not affected much by the occasional collision of a couple stars. This picture changes dramatically, however, as the gravothermal collapse progresses. When the core has shrunk down to a size of $\sim$ 0.1 pc, the local density has gone up to a whopping $\sim 10^7 \mathrm{pc}^{-3}$. This changes the mean time between physical encounters from the half-mass radius value of more than 10^{14} yr to a threatening 10^9 yr, significantly less than the age of a globular cluster.

There are other arguments pointing to the importantce of physical encounters. Even though the average star does not have much of a chance to have a physical collision before core collapse, the number of stars which have undergone an encounter close enough to lead to a tidal capture steadily increasing to a significant fraction of a percent. We do not know whether such near-misses, with a pericenter distance of less than about three stellar radii, can lead to the formation of close binaries, or whether the stars will spiral in to form a merger remnant (see below). But if they avoid merging, the encounters could result in the presence of $\sim$ 1000 close binaries in or near the core, just before core collapse (Statler *et al.* 1987). The presence of a number of binaries this large will certainly affect the evolution of the cluster, and models based on the point-mass approximation will not any longer give a reasonable approximation to the dynamics of the central area.

Finally, the outcome of three-body scattering events are not well represented by point-mass dynamics, even apart from the question of the feasibility of tidal capture. If we start with a moderately hard binary, say with an energy of $10kT$, in which both stars are on the main sequence, we have a semimajor axis for the orbit of order 1 A.U. – about a hundred times larger than the sum of the radii of the two stars. When a third star gets involved in a strong scattering encounter with such a binary, there is a large chance that this star will be temporarily captured. In such a resonance scattering event, many close encounters may take place, and in half of all cases the closest encounter will be at less than 3% of the original semimajor axis (Hut & Inagaki 1985). Therefore, our $10kT$ star already has a nonnegligible chance for a physical encounter, about 15% as it turns out. A somewhat harder binary of $30kT$ has a probability of 50% to suffer a collision during a resonance scattering, and the probability tends to unity for binaries which are much harder.

The situation is even worse for binary-binary scattering. Even a small abundance of primordial binaries will result in a much larger concentration in the core, due to mass segregation, and a proportionally enhanced probability for binary-binary encounters. In a typical resonance scattering event between two identical binaries, the typical closest encounter distance is only 1% of the original semimajor axis (Leonard 1991). Thus even an encounter between two $10kT$ binaries has already a fifty-fifty chance to result in a physical collision, with the chance become larger for a greater degree of hardness.

From all this it is clear that we have to take finite-size effects seriously before we can trust any detailed simulation enough to compare it reliably to observations of globular clusters. The next subsections give an overview of recent work in modeling finite-star-size effects.

6.2. Tidal Capture: Semi-Analytic Results

After the discovery of X-ray sources in globular clusters, the mechanism of tidal capture was suggested by Fabian *et al.* (1975) to explain the origin of these sources as the capture of a neutron star by a main-sequence star through the dissipation of the excess orbital energy in the tidal bulges raised on the normal star by the neutron star. They estimated the amount of energy dissipation by assuming that the energy needed to raise the tidal bulges would be dissipated efficiently (*cf.* Verbunt & Hut 1987).

More detailed calculations, for a variety of stellar models, were performed by Press & Teukolsky (1977), Lee & Ostriker (1986), Giersz (1986), McMillan *et al.* (1987), and Ray *et al.* (1987). These calculations mostly concentrated on the processes taking place during the first encounter between the two stars. They left open the question of what will happen during subsequent encounters.

In order to sketch the problem, let us start with an incoming star which brings in an excess amount of kinetic energy equivalent to that contained in its motion at infinity at, say, 10 km/sec. The amount of orbital energy transfer is highly dependent on the distance of closest approach. Therefore, most encounters will either not transfer enough orbital energy, or transfer far more than the relative kinetic energy at infinity. For definiteness, however, let us assume that the orbital energy lost is only twice the energy at infinity. As a result, the incoming star will be captured on an orbit with a binding energy corresponding to a circular orbital speed of order 10 km/sec, which implies a semimajor axis of order 10 A.U. Of course, the orbit will not be circular but highly elongated, and after some tens of years the two stars will meet again for a second close encounter.

If we can assume that the energy which was transferred in the first encounter to the tidal bulges has been dissipated, and if the star or stars in which tidal bulges were raised are not swollen significantly by the process of dissipation, then it is reasonable to assume that a similar amount of orbital energy is lost again during the second pass. If we can continue these assumptions for subsequent passages as well, we have the following scenario. If the Sun would be one of the two stars, the first capture orbit will stretch out to somewhere near Saturn's orbit, the second to Jupiter's orbit, the fourth to Mars' orbit, the sixth to that of the Earth, the eighth to Venus' and the tenth to Mercury's orbit. However, the last nine orbits will take less time to traverse than the first one, and within a hundred years the orbit is already enclosed within that of Mercury (meanwhile, the poor planets may have made off to infinity; good thing the Sun does not live in the core of a globular cluster).

There is a problem, though, with the two 'if's' involved in the previous paragraph, as was pointed out by Kochanek (1991). Perhaps the energy in the tidal bulges can be dissipated during the first passage, if the first captured orbit is wide enough. But it is not clear how the star(s) can get rid of the excess heat on the subsequent orbital timescales, of order of a few years or less. To shed more light on the nature of these problems, Kochanek developed a very interesting technique based on the affine star model introduced by Carter & Luminet (1985). Although his model has some limitations (radial modes and $\ell = 2$ f-modes only), it has two great advantages, stemming from the fact that the model is not based on a linear mode analysis approximation. First, it can be applied to any amount of distortion, and is therefore accurate for arbitrarily close encounters (until the star flies apart, literally). Second, the dissipation is based on an actual dynamical model of a star, which can be followed from one close passage to the next.

Kochanek's study reached several interesting conclusions. Most importantly, he showed explicitly that our intuition of interacting binary evolution cannot be applied to that of tidal capture, simply because the latter takes place on an orbital time scale while the former generally happens on a thermal time scale (apart from episodes of unstable mass transfer or catastrophic mass loss). As a result, the evolution of the capture orbit will be highly stochastic, depending on the exact phases of the main modes of oscillation of the star(s) during each subsequent pericenter passage. While the orbital energy and angular momentum are most likely to decrease on average, individual encounters can lead to either a gain or a loss. The simplified picture sketched above thus turns out to be completely inaccurate.

Kochanek concludes that the large majority of close encounters probably cannot lead to tidal capture, due to a variety of problems, such as the possibilities of a stellar disruption or extensive mass loss, of a physical collision, or escape due to a third passing star. Surprisingly, those binaries that do survive may well have an orbit significantly wider than had been estimated by earlier authors. He estimates that a few percent of the encounters lead to orbits with a pericenter distance of more than 50 stellar radii, due to perturbations of a third star, which can transfer extra angular momentum to the binary. If the large majority of all other cases lead to some catastrophic development or other, these unusually wide orbits may make up a large fractions of those orbits which survive.

6.3. Hydrodynamical Effects: Numerical Results

Detailed numerical simulations of the hydrodynamical effects occurring in close encounters between stars have only recently become feasible. The main problem is that the combination of the stellar rotation and the orbital revolution makes a fully three-dimensional treatment absolutely necessary. Fortunately, computer speed and availability have dramatically increased over the last few years, to the point that the initial encounter of two or more stars can now be modeled with at least some confidence that the major features of the calculations correspond to reality. The question raised by Kochanek (1991), of what happens after successive encounters, is still completely beyond present computational reach, unfortunately. Waiting for orders of magnitude of increase in raw computer speed would be frustrating. Perhaps a clever and careful development of new algorithms will bring such calculations within the realm of possibilities. A combination of fully 3-D hydro calculations during close encounters, together with semi-analytical models which can follow the approximate damping of the oscillations during the intervening times, may be one approach.

After the first explorations by Benz & Hills (1987) a number of 3-D hydro treatments of close stellar encounters have appeared during the last couple years, by Benz *et al.* (1989), Cleary & Monaghan (1990), Goodman & Hernquist (1991), Davies *et al.* (1991) and Rasio & Shapiro (1991). A discussion of each of these papers is beyond the scope of the present review. A general pattern emerging from these calculations has been the very efficient merging of stars involved in scattering processes which lead to close encounters in which the outer layers of the stars began to overlap. In the treatment of binary-binary encounters by Goodman & Hernquist for example, multiple mergers were common in which three or even all four stars fused. Obviously, outcomes like this will have a very significant effect upon the outcome of detailed N-body simulations of globular clusters.

How will we be able to incorporate these hydrodynamical effects into the star-by-star simulations which will become available, several years from now? There is one piece of good news, together with the bad news that point-mass simulations of star clusters already will require Teraflop speeds (Hut *et al.* 1988). The good news is that the addition of a modest amount of hydrodynamic modeling of close encounters will not significantly affect the total computational cost! This follows from the analysis by Makino & Hut (1990), who showed that the relative cost of computations involving binary encounters decreases $\propto N^{-25/12} \log N$, or roughly $\propto N^{-2}$, as compared to the overall computational cost of following the orbits of the single stars. Thus the binary cost drops steeply for increasing N, even though the ratio of the half-mass crossing time to the typical binary orbital time increases $\propto N$.

There are two reasons for this counter-intuitive result: on a local level, very tight binaries can be modeled as isolated point masses for an increasing fraction of the time for increasing N; while on a global level, the rate of formation of binaries per half-mass crossing time is ~ 0.1, independent of N. The detailed analysis by Makino & Hut showed that a typical binary requires a total of $\sim 10^7$ time steps to integrate the relative motion of its components, during its whole life. In contrast, the computational cost of the system as a whole increases strongly, $\propto N^2$, even per half-mass crossing time. They conclude that there are three regimes in N-body simulations of star clusters. For $N < 1000$, binaries form the computational bottleneck, requiring more computer time than the single stars. For $1000 < N < 10,000$, the fraction of computer time needed for binary integration drops steeply, from ~ 1 to $\sim 10^{-4}$. For $N > 10,000$, it becomes feasible to model stellar collisions and close encounters using Smooth Particle Hydrodynamics, using at least a few hundred particles per star, or even $> 10^4$ particles per star for $N > 10^5$. The implementation of a SPH code inside a N-body code will be far from trivial, though (see also §5.7).

A realistic simulation of globular cluster evolution should include stellar evolution effects as well, both for single stars as well as binaries. Including stellar evolution into an N-body code does not increase the computational cost in any appreciable way. The integration of a typical star, including the giant stage, requires a few hours on a VAX 11/780, which corresponds to $\sim 10^9$ floating point operations. Even if all 10^5 stars needed such treatment, the total cost would be $\sim 10^{14}$ floating point operations, which is much smaller than that required for the whole star cluster in the point-mass approximation (Hut *et al.* 1988). Thus from a hardware point of view, inclusion of stellar evolution is trivial. Of course, to develop the software needed, by integrating some form of stellar evolution code into an N-body code, will be a major undertaking, already for single-star evolution, and much more so for binary star evolution, where problems such as spiral-in in a common-envelope phase will occur.

7. Binary Migration

Giants and subgiants in globular clusters all have a mass comparable to that of the turn-off mass, around $0.8M_\odot$, as a consequence of sharing the age of the clusters, on the order of 10^{10}yr. Main-sequence stars, which make up the bulk of the mass of a cluster, are less massive than that. Some of the white dwarfs are probably heavier than $0.8M_\odot$, up to $1.2M_\odot$, but the total mass in these stars is unlikely to exceed a few percent of the total cluster mass, and may be much smaller (*cf.* Murphy *et al.* 1990). Neutron stars, around $1.4M_\odot$, are even rarer. Merger remnants are likely to be present too, but also in relatively small numbers (see §6.1).

Primordial binaries tend to be more massive than single stars, on average, simply because each member star lives under the same constraints as a single star, its mass being bounded above by the turn-off mass. As a result, primordial binaries will drift to the central regions of the cluster. This mass segregation is analogous to the effects in the Earth's atmosphere, where heavier gasses, such as CO_2, have a smaller scale height than the lighter gasses. A factor two difference in mass, on average, is enough to concentrate most binaries within the inner few core radii, in the final equilibrium distribution. This distribution can be described conveniently using a multi-mass generalization of King models. Indeed, this approach has been widely taken in the analysis of cluster observations.

The main question is then: will this thermodynamic equilibrium ever be reached? McMillan *et al.* cautioned against the assumption of equilibrium, by pointing out how the recoil of binary–binary and binary–single-star scattering events tend to place many binaries on wide 'halo parking orbits' for a considerable length of time. They observed a bimodal distribution of recycled binaries (as opposed to the pristine primordial binaries still drifting in from the far halo, even at late times): at any given time, a snapshot of their simulations revealed that most of the binaries which had passed at least once through the core could be found in or near the core, but some would be positioned far out in the halo, with relatively few in the intermediate area.

Similar conclusions were reached by Phinney & Sigurdsson (1991), who applied this reasoning to the distribution of the pulsars in M15. Specifically, they argued that the binary pulsar 2127+11C is an example of a binary currently parked in the halo, since the projected distance from the center is ~ 20 core radii (*cf.* the contribution by Phinney in the present proceedings).

Since the binary distribution through the cluster has obvious observational interest, it is really frustrating that state-of-the-art cluster simulations are not able to offer clear predictions. The problem is that Fokker-Planck models cannot handle recoil (*cf.* §4.3), and that N-body models cannot handle a large enough particle number (*cf.* §4.2). While the final solution will be to wait for N-body modeling to catch up, Hut *et al.* (1991b) decided to take a short-cut, by modeling the evolution of individual binaries in a fixed background potential determined by the single star distribution (a similar approach is being developed by Phinney and Sigurdsson; *cf.* the contribution by Phinney in the present proceedings).

In the model by Hut *et al.* binaries are characterized by a two-dimensional distribution function in binding energy and distance to the cluster center (or, equivalently, internal and external orbital energy). The model forms a middle ground between realistic N-body calculations and analytic estimates based on average amounts of binary hardening per scattering event. They present a series of Monte Carlo simulations for an initial population of 5×10^4 binaries against a fixed background population of 5×10^5 single stars in a tidally truncated cluster model. They followed the individual histories of all binaries as they experience a variety of different physical mechanisms: mass segregation, scattering recoil, escape from the cluster, and, optionally, coalescence through gravitational radiation losses and collisional mergers.

The main observational consequences of their simulations are: 1) most binaries are destroyed by binary–binary interactions. In the point-mass approximation, the rest escape. In a more realistic model, the majority of the rest merge. 2) At any instant, most binaries are drifting in toward the center, before their first strong encounter. 3) A typical binary spends most of its active life (after their first strong scattering event) in or near the cluster core. 4) The few binaries which receive a recoil sufficient to place them in the halo past the half-mass radius remain there long enough to make a significant contribution to the radial binary distribution. 5) This latter effect is strongly suppressed by collisions and spiral-in, both of which tend to lower the average distance of a binary from the cluster center.

It was an exciting exercise, to see how a population of 50,000 binaries evolved from scratch. It certainly whets one's appetite for a self-consistent N-body simulation, in which similar numbers of stars can be followed with a dynamically responding core. But even the relatively crude estimates following from the fixed-background simulations are already providing a lot of incentive to test our qualitative predictions. Fortunately, rapid progress is being made in the observational techniques for finding binaries in globular clusters, as summarized by Hut *et al.* (1992).

8. Galactic Nuclei

Much of what has been discussed so far can be applied not only to globular clusters, but to at least some galactic nuclei as well. For many galaxies, the velocity dispersion in the central regions is an order of magnitude larger. As a consequence, processes involving binaries are affected significantly. For example, physical collisions become more likely for hard binaries, since the member stars have to be closer. Also, gravitational radiation plays an important role in binary destruction at such small separations. Nonetheless, there are many interesting areas of overlap between the evolution of globular clusters and of galactic nuclei. The first meeting specifically dedicated to a comparison between these two areas was held a few years ago at CITA. Pointers to the literature can be found in the proceedings of this meeting (Merritt 1989), especially in the contributions by Murphy *et al.* (1989), Lee (1989), Quinlan & Shapiro (1989), and Rasio *et al.* (1989). More recent work was published by Quinlan

& Shapiro (1990) and Murphy *et al.* (1991).

Not all galactic nuclei have a velocity dispersion which is much larger than that typical for globular clusters. An interesting exception is the spiral galaxy M33, a member of the local group. Recent observations by Kormendy & McClure (1991) give a central line-of-sight velocity dispersion of 22 km s^{-1}, only twice that in a typical globular cluster. Their observed limit on the core radius is $r_c \simeq 0.3$ pc. From these observations, Hernquist *et al.* (1991) argue that the true core radius is likely to be $r_c \lesssim 0.1$ pc.

In short, their arguments run as follows. Using the virial theorem, or more specifically, the model of an isothermal sphere (Binney and Tremaine 1987, eq. (4-123)), the local density $\rho(0.3\,\mathrm{pc}) = 2.0 \times 10^5$ $M_\odot$ pc^{-3}. The corresponding local two-body relaxation time (Binney and Tremaine 1987, eq. (8-71), and Spitzer 1987, eq. (2.62) with $m = M_\odot$ and $\ln \Lambda = 10$) is $t_{\mathrm{relax}}(0.3\,\mathrm{pc}) = 9.4 \times 10^7$ y. Since 0.3 pc is an upper limit to the core radius and since the central density is about three times that at the core radius, the central relaxation time is $t_{\mathrm{relax}}(0) \lesssim 3 \times 10^7$ yr. Such a short relaxation time implies that the system probably has already undergone core collapse.

With the observed velocity dispersion of 22 km s^{-1}, Hernquist *et al.* suggest that a collapsed core may have a true core radius $r_c \simeq 0.1$ pc. They offer three independent arguments for this value. First, simulations without primordial binaries show that cores oscillate after core collapse and that they spend most of their time near maximum expansion. They then typically contain 0.5 % to 1% of the mass of a globular cluster. The corresponding core radius is $r_c \sim 0.1$ pc (Murphy *et al.* 1990). Second, a significant population of primordial binaries will cause core collapse to halt and reverse at relatively large core radii. Again the value is $r_c \sim 0.1$ pc (Goodman & Hut 1989, McMillan *et al.* 1990, 1991, Gao *et al.* 1991). Third, HST observations show that M15 has a core radius of $r_c \sim 0.1$ pc (Lauer *et al.* 1991).

Hernquist *et al.* conclude that the nucleus of M33 has an efficiency of low-mass X-ray binary formation equal to that of all Galactic globular cluster cores combined. This suggests that about a dozen such binaries should be present. Their combined emission may explain the enigmatic, unresolved X-ray source in M33's nucleus, with its high X-ray luminosity of 10^{39} erg s^{-1} (Long *et al.* 1981). Besides X-ray binaries, they also predict a large population of $\sim$ 100 ms pulsars.

9. Discussion and Outlook

This review has focused on the many new directions which have been taken in the modeling of globular clusters over the last few years. There are a number of exciting developments, both in the simulations of the evolution of globular clusters as complete systems, as well as in the treatment of the microphysical processes: close encounters and collisions between individual stars.

The most exciting development lies in the possibility to compare the simulations with observations, both before and after core collapse. This is a sign that the simulations have finally reached a degree of maturity which had been absent in the first couple decades of modeling. A milestone in this respect has been the ability to model the post-collapse evolution of a globular cluster using a realistic mass spectrum (Murphy *et al.* 1990, Lee *et al.* 1991). The first examples of an application of these simulations to the real world have been given by Grabhorn *et al.* (1991), who model the evolution of the globular clusters M15 and NGC 6624, and fit the observed surface-brightness and projected velocity dispersion profiles. They find good agreement with simulations in which the core size is near the maximum value attained during oscillations.

Interesting as these developments are, it is also important to realize the many shortcomings and unwarranted approximations which are still present in the present state-of-the-art simulations, both in Fokker-Planck models which have difficulty handling binary fluctuations as well as in N-body models which cannot yet handle large-N values (see §§4.2-3). Whenever Teraflop speeds will become available, these limitations will disappear. A promising development in this direction is the GRAPE project at Tokyo University (see §4.2), aimed at developing gravitational-force-accelerators in the form of chips which can be added to workstations to reach multi-Gigaflop speeds.

Even when Teraflop speeds will be routinely available, there will be additional bottlenecks in our understanding of globular cluster evolution. The main remaining problem will then be the modeling of the micro-physical processes which occur during scattering events involving close encounters between single stars and binaries. Three-dimensional hydrodynamical modeling of such events is now beginning to be feasible on the dynamical timescale of a single close encounters. Unfortunately, we do not yet know of any good approach toward modeling the longer-term behavior of the oscillations, mass loss and break-up of stars involved in the three-body and four-body dances triggered by these scattering events, which may last many thousands of years (see §6.2). Nonetheless, it would be a very useful start to be able to run a star-by-star modeling of a globular cluster with a simple hydrodynamical treatment of close encounters, using for example a 1000-particle SPH model for each star involved in such an encounter, for the duration of the encounter. Such calculations should become possible before the end of the decade.

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