1 Homework I

1. Eigenvectors and eigenvalues

Let

$$K = \begin{pmatrix} .75 & .25 \\ .25 & .75 \end{pmatrix}.$$

- Show that 1 and 1/2 are eigenvalues of K and find the eigenvectors. Express K as PDP^{-1} where D is diagonal and P is orthonormal.
- Let $K' = \alpha K$ for real $\alpha \neq 0$. Find the eigenvalues and eigenvectors of K'.
- Consider the mth power of K, K^m, for $m > 0$. Find the eigenvectors and eigenvalues of K^m.

2. Gaussian random variables

Recall that a Gaussian random variable $\mathbf{x}$ with mean μ and standard deviation σ has density:

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

We will use the notation $\mathbf{x} \sim \mathcal{N}(\mu, \sigma)$.

- Suppose a and b are constants. Show that if $\mathbf{x} \sim \mathcal{N}(0,1)$, then $b\mathbf{x} + a \sim \mathcal{N}(a, b)$.
- Show that if $\mathbf{x}_1 \sim \mathcal{N}(\mu_1, \sigma_1)$ and $\mathbf{x}_2 \sim \mathcal{N}(\mu_2, \sigma_2)$, then $\mathbf{x}_1 + \mathbf{x}_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2})$.
- Suppose $\mathbf{x} \sim \mathcal{N}(0, \sigma)$. What is $\mathbb{E}(\mathbf{x}^2)$? What is the distribution of $\mathbf{x}^2$?