# SPARSITY:
# GENEARLIZED SPARSITY MEASURES AND APPLICATIONS

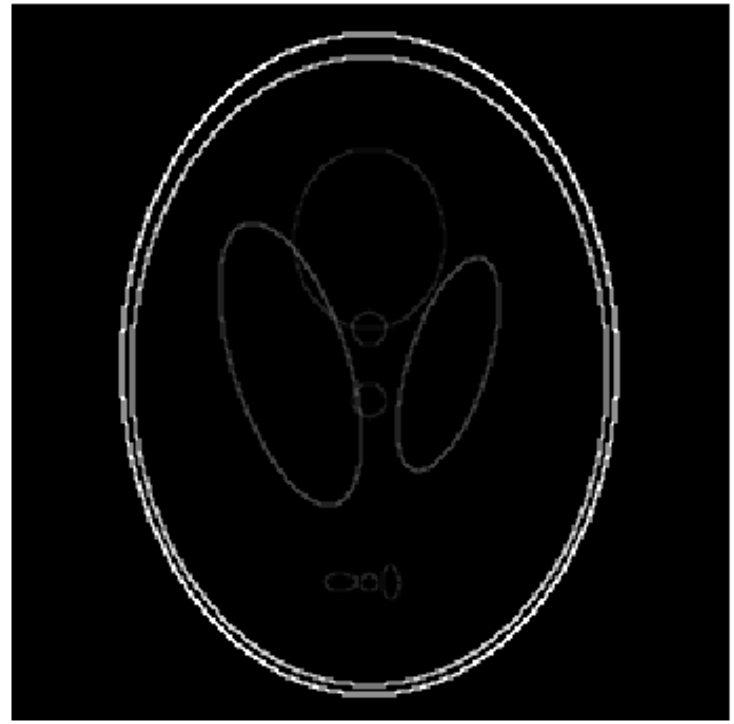
Rebecca Willett

DUKE
ENGINEERING
EDMUND T. PRATT, JR.
SCHOOL OF ENGINEERING

# Generalized Sparsity Measures

# TOTAL VARIATION

Image



Squared image gradient

- Sometimes an image's gradient is sparse
- There is not a good orthonormal basis representation of this
- Total variation approximately measures the image gradient

# Total Variation

$$\widehat{f} = \arg\min_{f} \|y - Rf\|_2^2 + \tau \|f\|_{\mathrm{TV}}$$

where

$$\|f\|_{\mathrm{TV}} \triangleq \sum_{i_1=1}^{N_1-1} \sum_{i_2=1}^{N_2-1} |f_{i_1,i_2} - f_{i_1+1,i_2}| + |f_{i_1,i_2} - f_{i_1,i_2+1}|$$

or

$$\|f\|_{\mathrm{TV}} \triangleq \sum_{i_1=1}^{N_1-1} \sum_{i_2=1}^{N_2-1} \sqrt{|f_{i_1,i_2} - f_{i_1+1,i_2}|^2 + |f_{i_1,i_2} - f_{i_1,i_2+1}|^2}$$

This approach produces state-of-the-art results in many settings, but there is relatively less theoretical support.

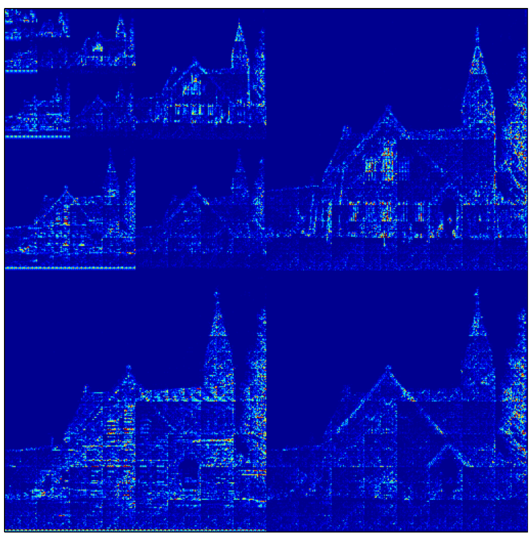
Rudin, Osher, Fatemi, 1992

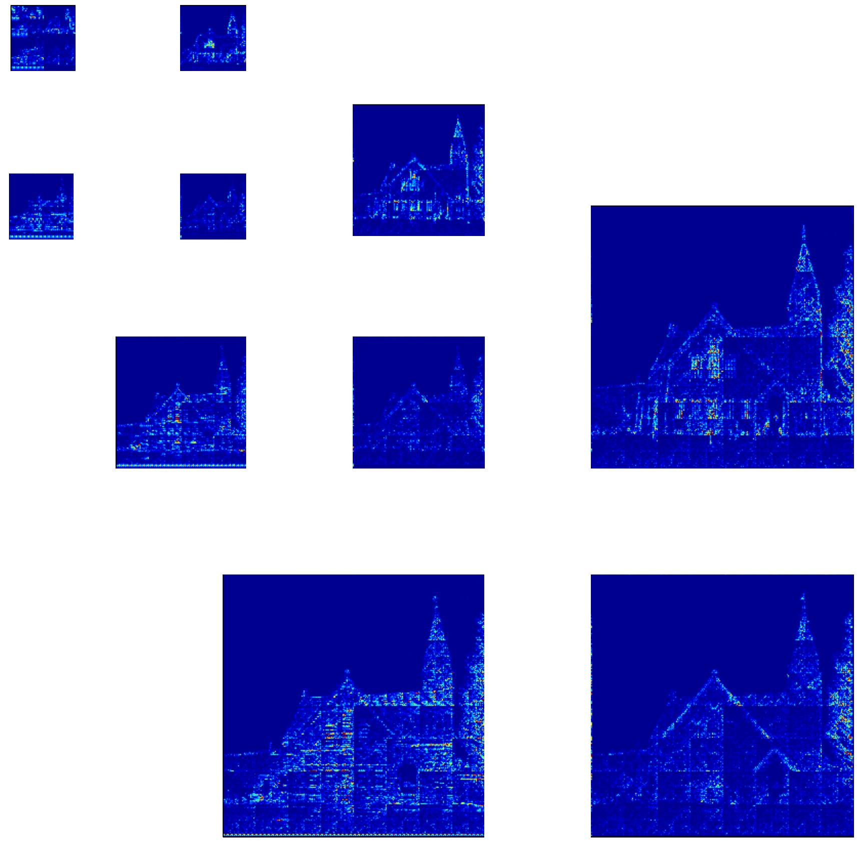
# STRUCTURED SPARSITY

- Until now, we have focused exclusively on sparsity, but have not assumed any additional structure.
- However, we often understand more about structure in signals and images than spasity alone…

Baraniuk, Cevher, Duarte & Hegde, 2010

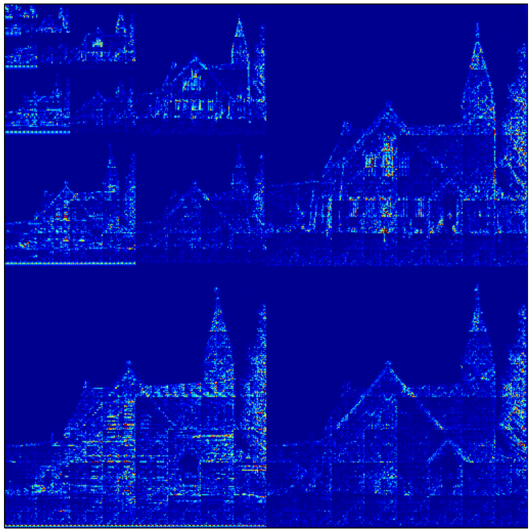
# Structured Sparsity

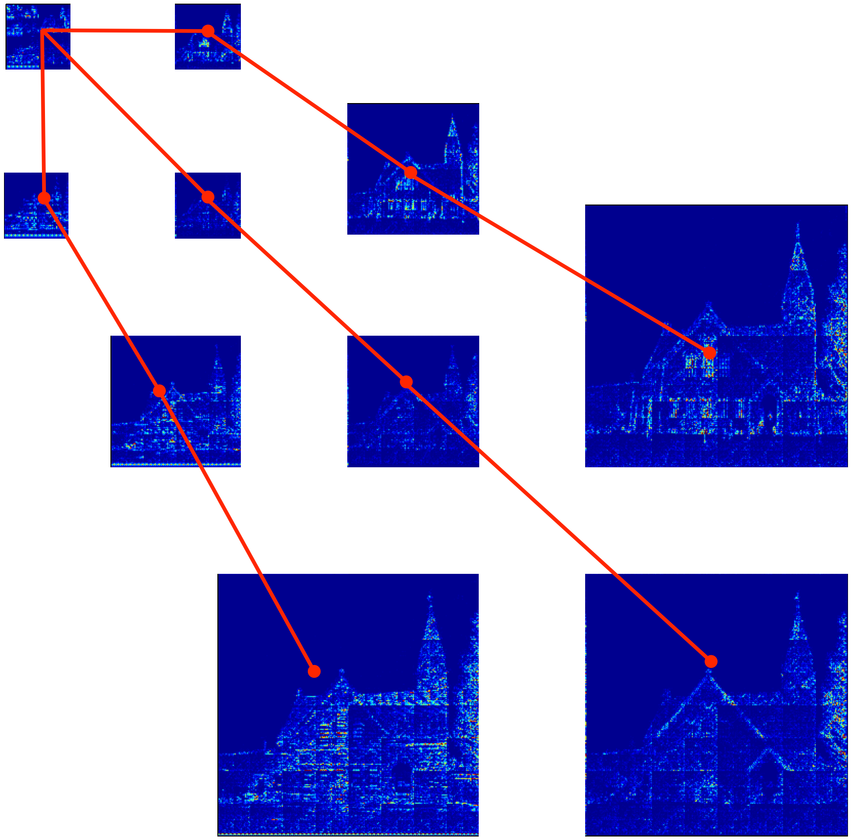




Baraniuk, Cevher, Duarte & Hegde, 2010

# STRUCTURED SPARSITY



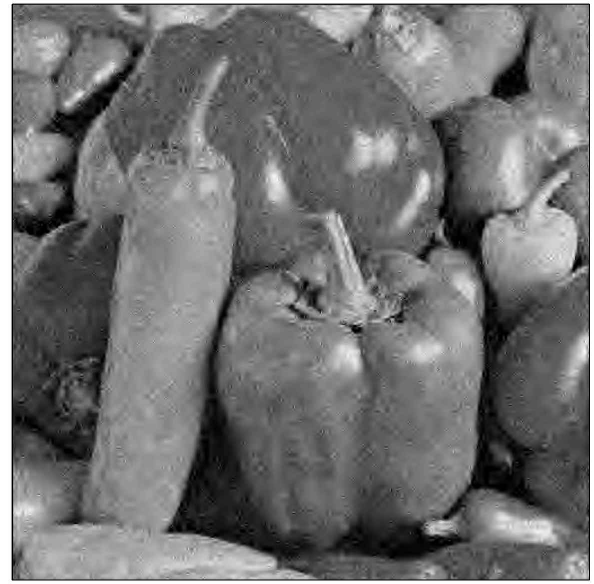


Baraniuk, Cevher, Duarte & Hegde, 2010

# STRUCTURED SPARSITY RESULTS

Original image

Standard CoSamP
PSNR = 19.9dB



**Structure-aware CoSamP**
**PSNR = 26.8dB**

Baraniuk, Cevher, Duarte & Hegde, 2010

# Block sparsity

This rich image is not especially sparse in "usual" bases such as a wavelet basis

# Block sparsity

It does contain quite a bit of structure, however. In particular, many “patches” of pixels appear repeatedly throughout the image…

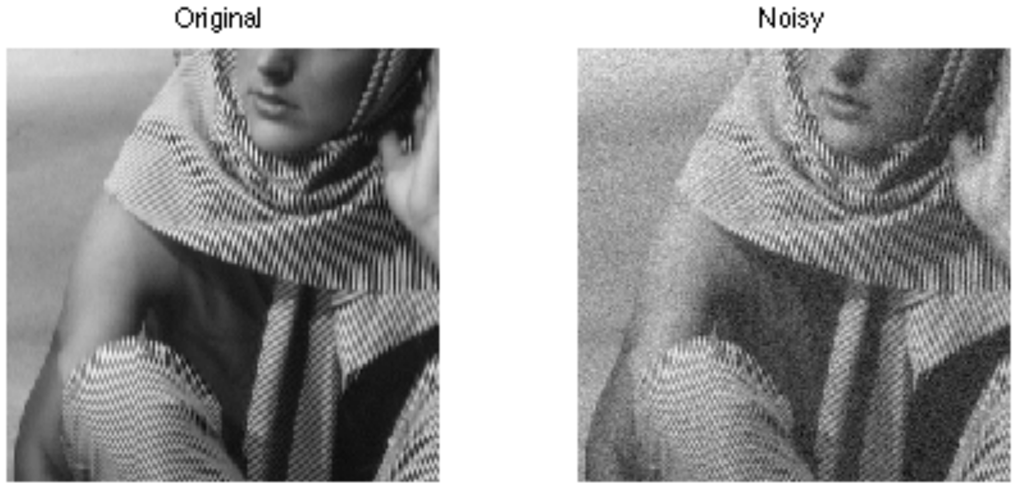
# Block sparsity

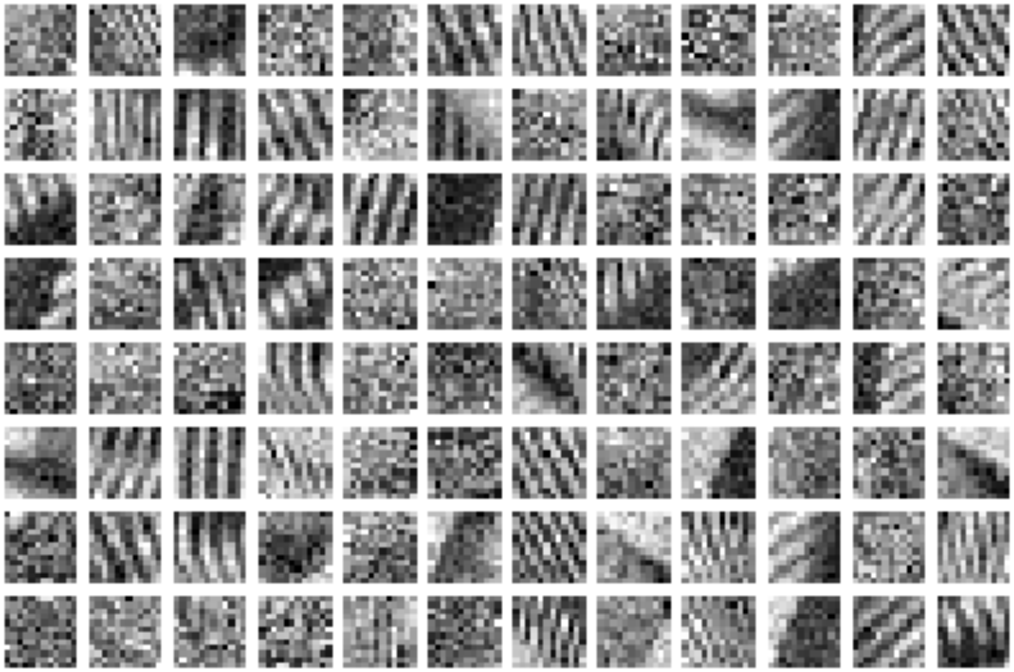
We can think of these patches as lying on a low-dimensional submanifold…

# Block sparsity

What if we could find a basis (or, more generally, dictionary) for patches so that all the patches are sparse in that basis?

# DICTIONARY LEARNING



# Dictionary Learning

First we extract all overlapping patches, $\{x_i\}_{i=1}^N$. Next we solve a matrix factorization problem:

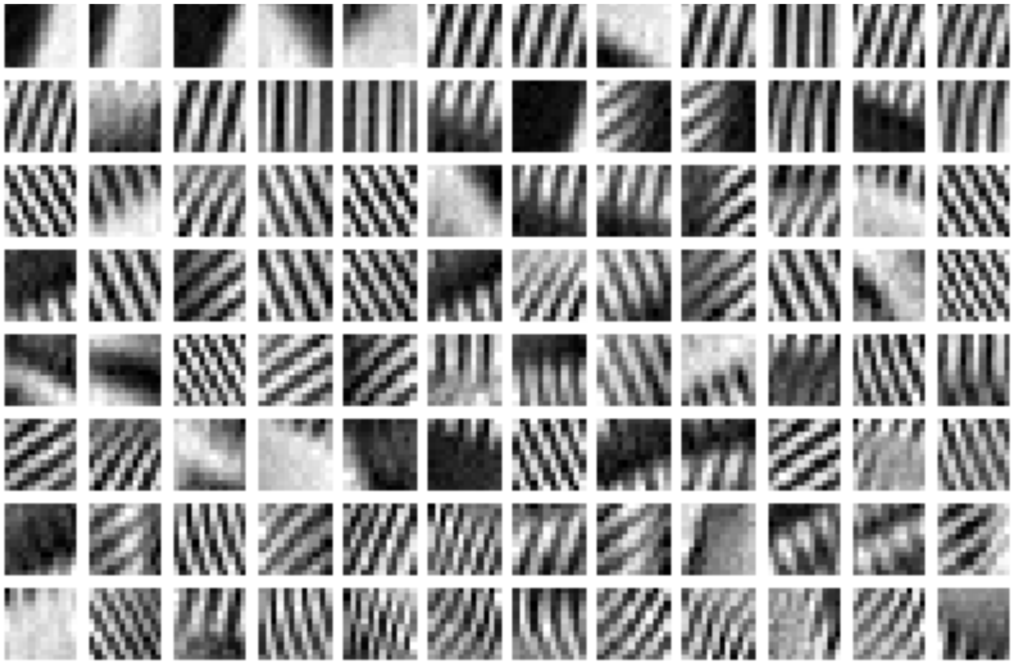
$$\min_{\alpha_i, D} \sum_{i=1}^{N} \|x_i - D\alpha_i\|_2^2 + \tau\|\alpha_i\|_1$$

- $x_i$ is the $i^{\text{th}}$ noisy patch
- $D$ is the dictionary
- $\alpha_i$ is the vector of dictionary coefficients for patch $i$

Once we have the dictionary $D$, we can denoise each patch in the dictionary ($\widehat{x}_i = D\alpha_i$) and re-form the denoised image.

Elad & Aharon, 2006

# Denoising with learning dictionary



Wavelet, SNR=23.33dB

Learning, SNR=24.73dB

# Dictionary Learning

Denoising result

Mairal, Bach, Ponce, Sapiro & Zisserman, 2009

# DICTIONARY LEARNING

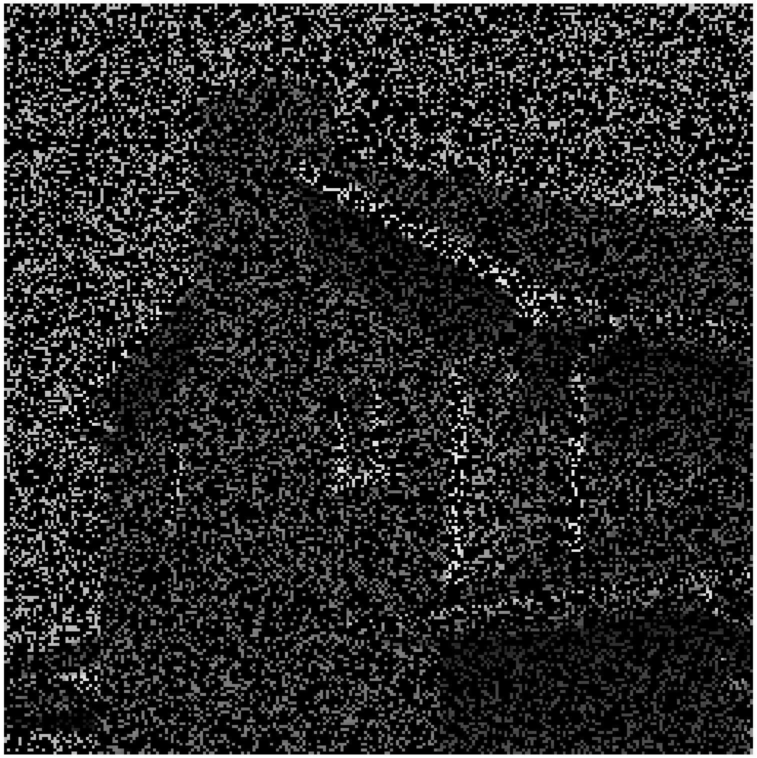


Image completion example

Mairal, Sapiro & Elad, 2008

# Compressive Coded Aperture Imaging

# KEY CHALLENGES

- Fast methods to compute $\widehat{f}$
- Good sparsity models for "natural" images
  - Images with boundaries
  - Images with texture
  - Hyperspectral images
- Incorporating real-world constraints
  - Photon noise
  - Non-negative intensities
  - Quantization effects
- Practical systems to measure compressed sensing data

# Challenge: Build imaging systems that exploit CS theory

- Projection $(A)$ with randomly-drawn elements not realizable in most optical systems:
  - Light intensities cannot be negative
  - Simultaneous projections require complex systems
  - Individual random projections taken at each time step not suitable for dynamic scenes
- Verifying the RIP for a particular matrix computationally intractable

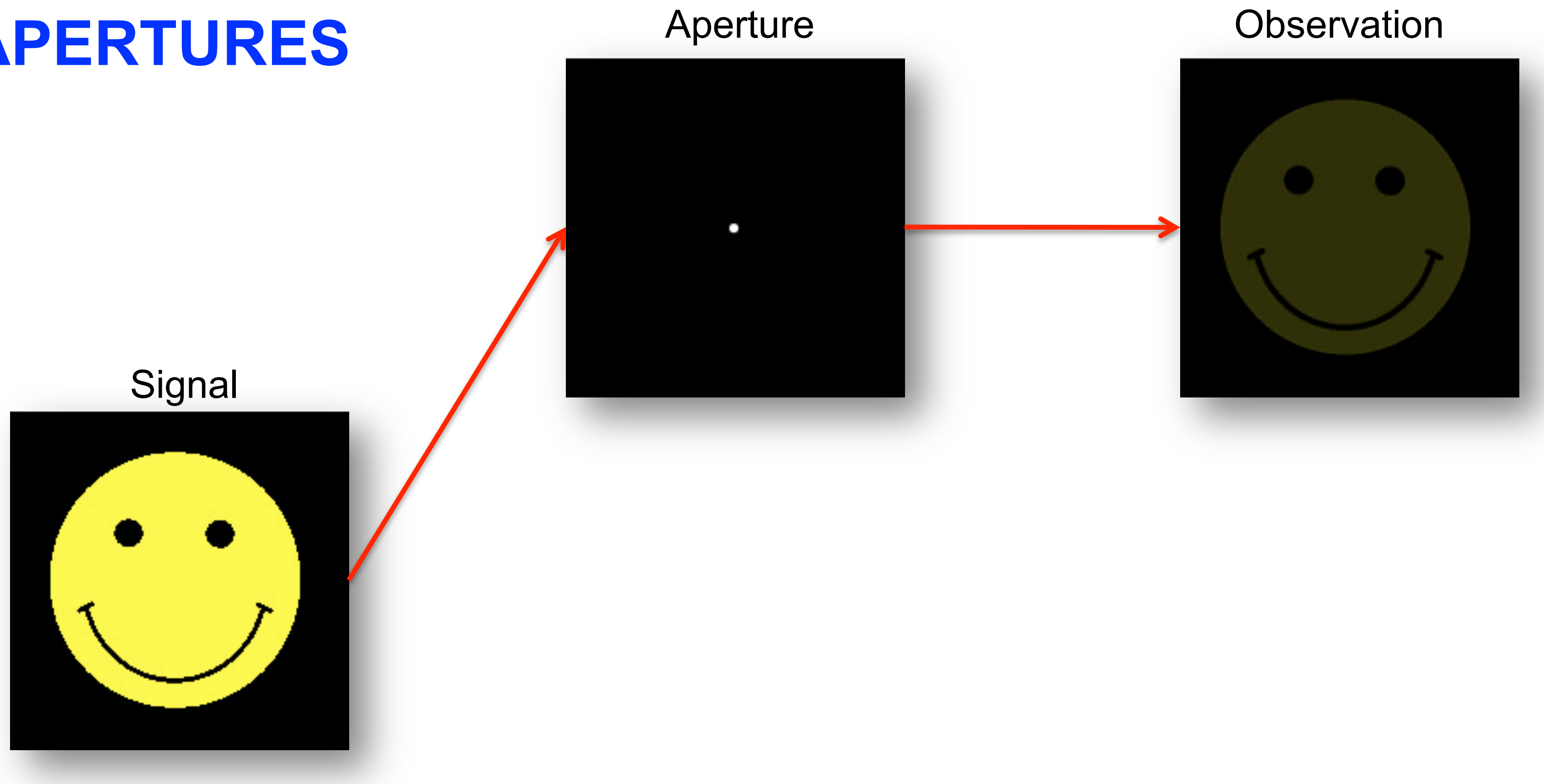
# Coded Apertures

- Simple to build and to incorporate into practical, compact optical designs
- RIP satisfied with high probability using CS theory for Toeplitz matrices
- Weaker theoretical guarantees

# APERTURES

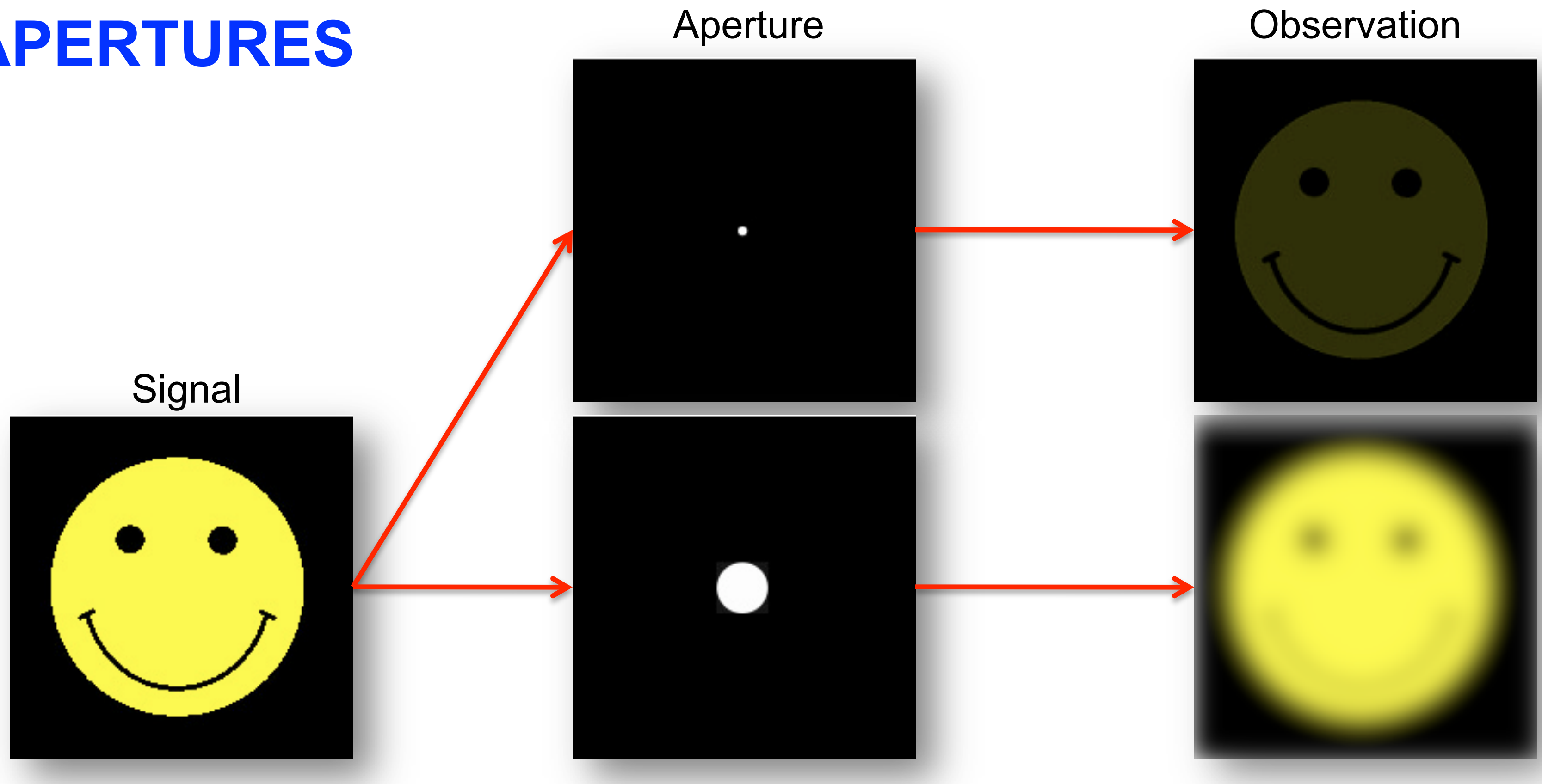
Signal

# APERTURES



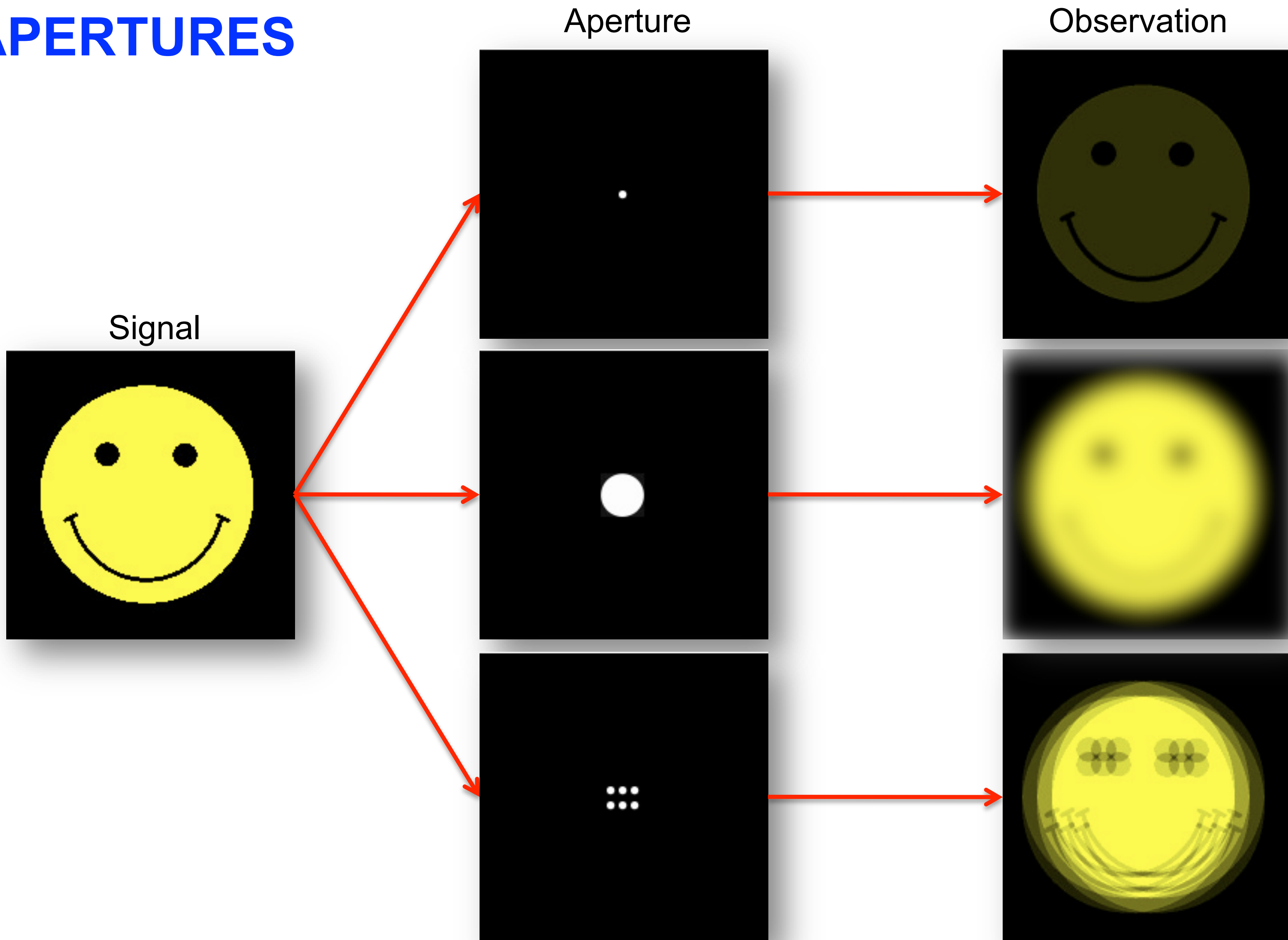

Small pinholes allow little light ⇒ dark observations.

# APERTURES



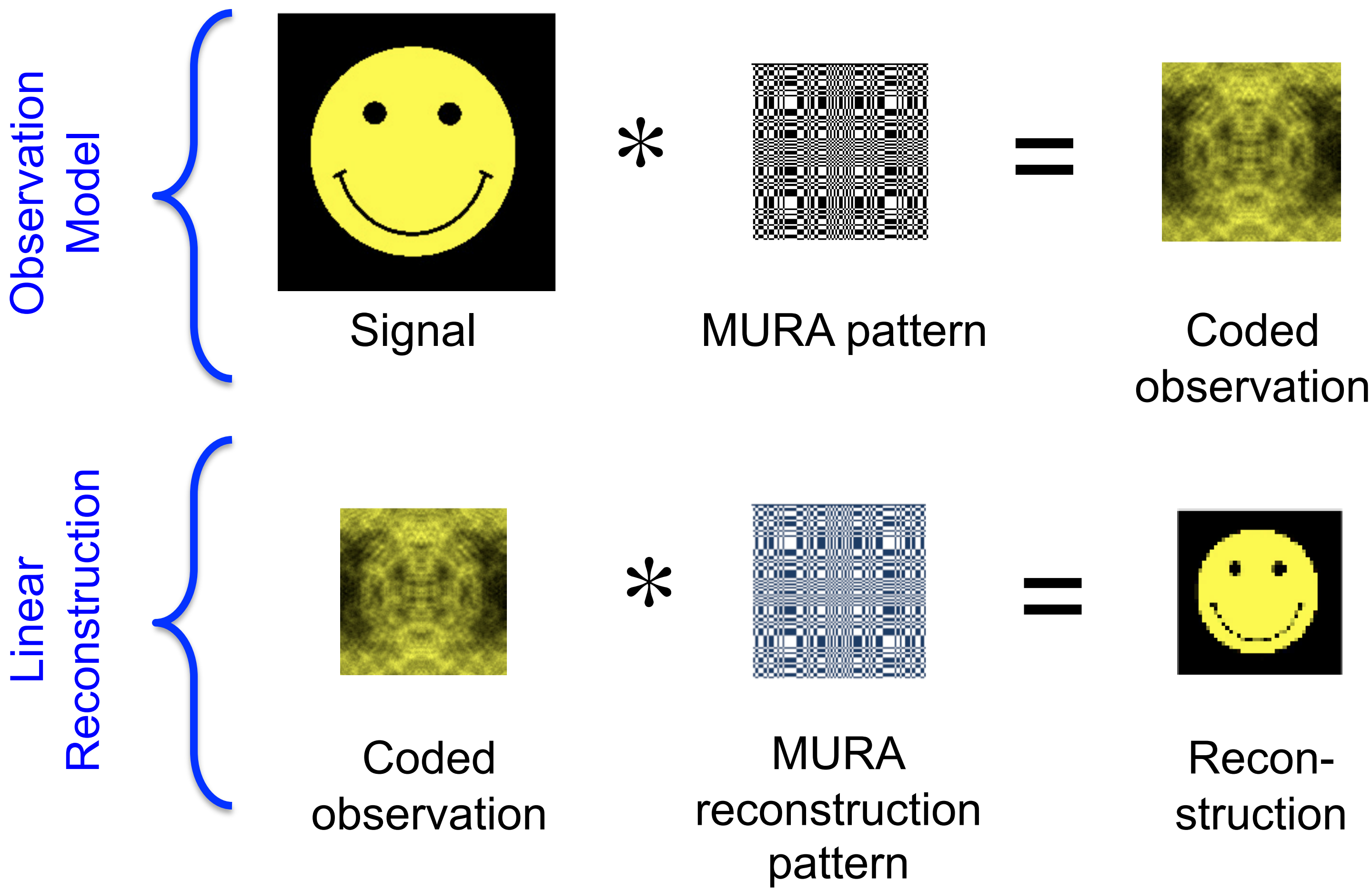

Larger pinholes allow more light but leads to decrease in resolution ⇒ blurry observations.

# APERTURES




Multiple small pinholes ⇒ overlapping observations.

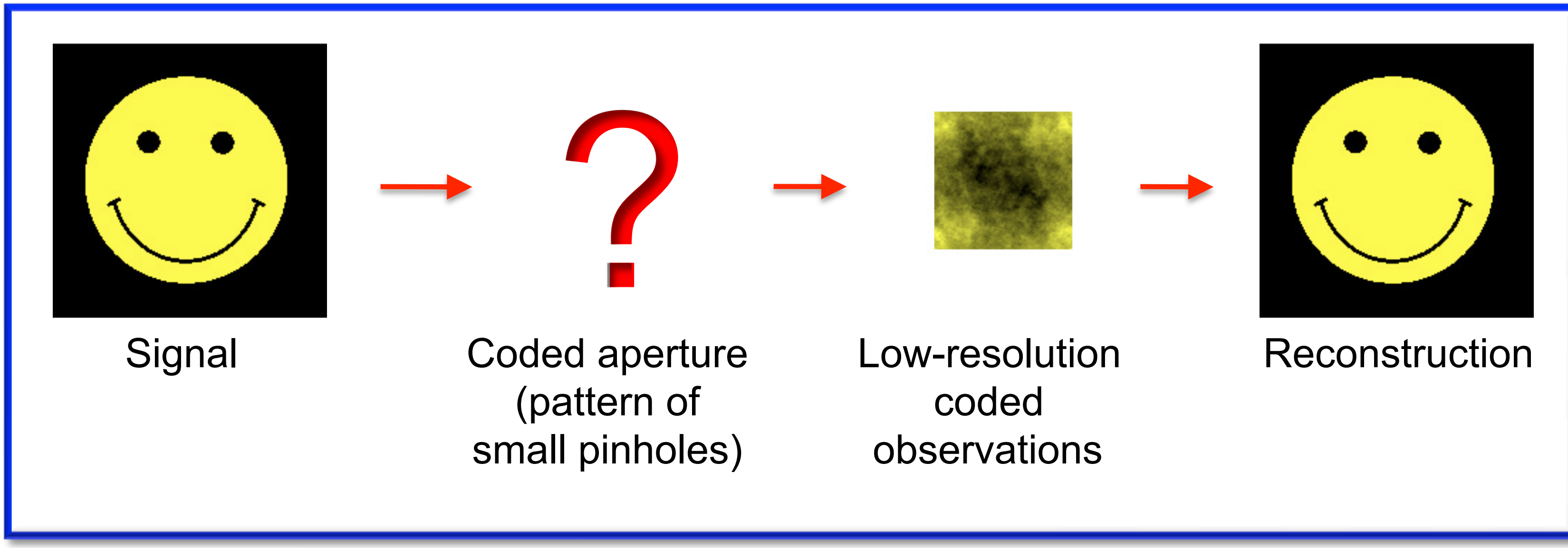
# Coded Aperture Imaging
## (Modified Uniformly Redundant Arrays)




**Resolution restricted to size of detector**

Gottesman and Fenimore (1989)

# Compressive coded aperture imaging




Observation model:

$$x = \overbrace{D(f * p)}^{A_p f} + \epsilon$$

Downsampling Operator

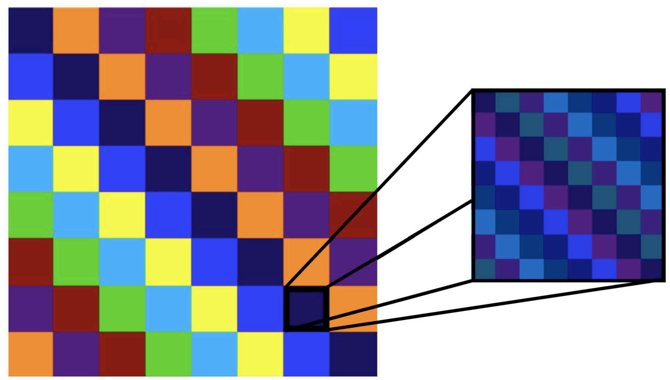
Signal

Coded Aperture

Noise

# COMPRESSIVE CODED APERTURE IMAGING

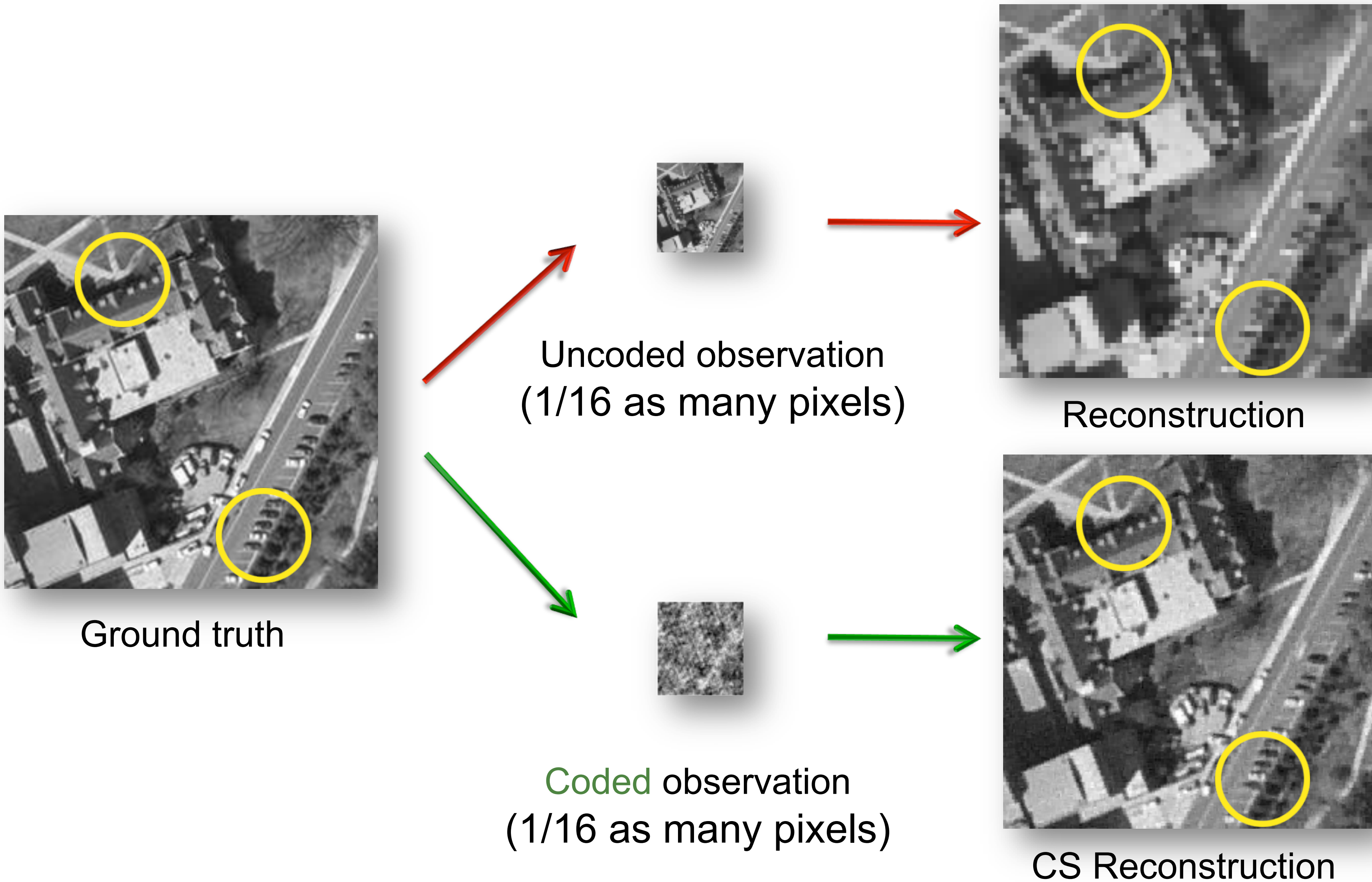
The sensing matrix $A_p$ is block circulant with circulant blocks:



**Theorem:** A mask pattern $p$ can be designed such that the resulting projection matrix $A_p$ satisfies a (weakened) RIP with high probability.

Marcia and Willett (2008a)
Romberg (2009)

# Compressive Coded Aperture: Video Example



# Reconstructed video (from 2-frame method)

Original Scene | Downsampled | Reconstruction from coded observation

# RECONSTRUCTIONS OF THE 25TH FRAME

Original Scene | Downsampled | Reconstruction from coded observation

# Reconstructed video (from 2-frame method)

Original Scene

Original Scene

Downsampled

Reconstruction from coded observation

# RECONSTRUCTIONS OF THE 25TH FRAME

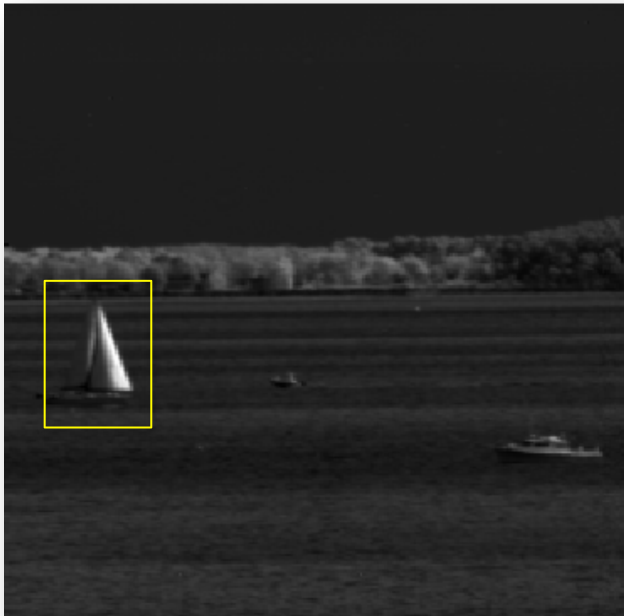
Original Scene

Original Scene

Downsampled

Reconstruction from coded observation

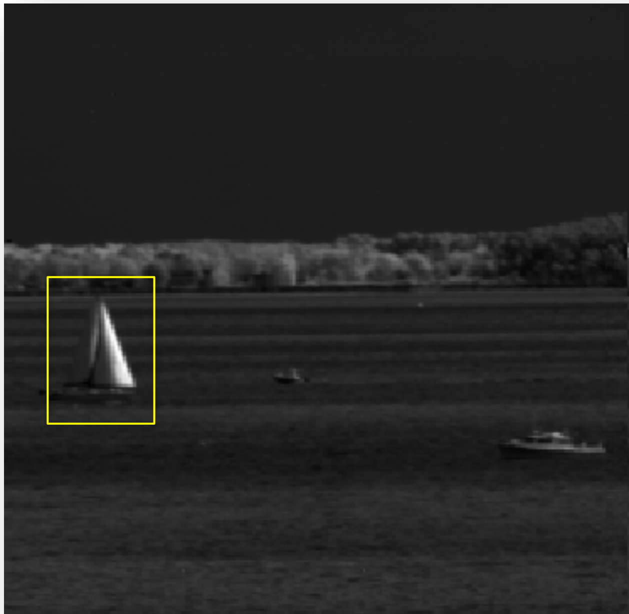
# RECONSTRUCTED VIDEO (FROM 2-FRAME METHOD)



Original Scene

Original Scene    Downsampled    Reconstruction from coded observation

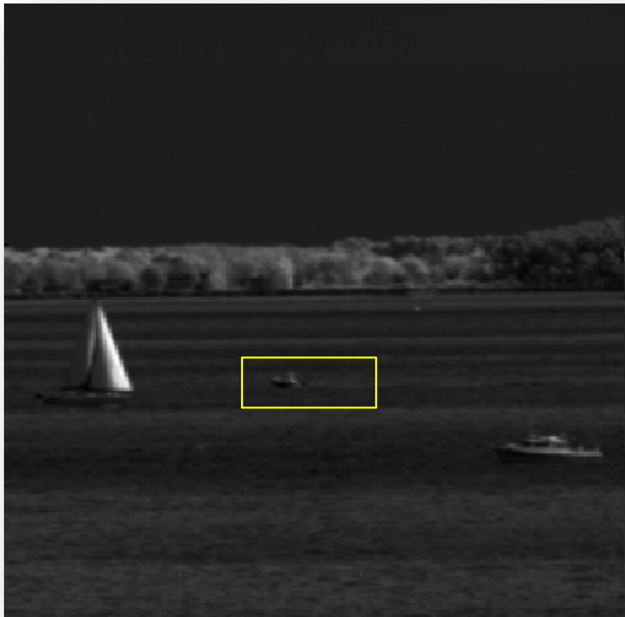
# Reconstructions of the 25th frame



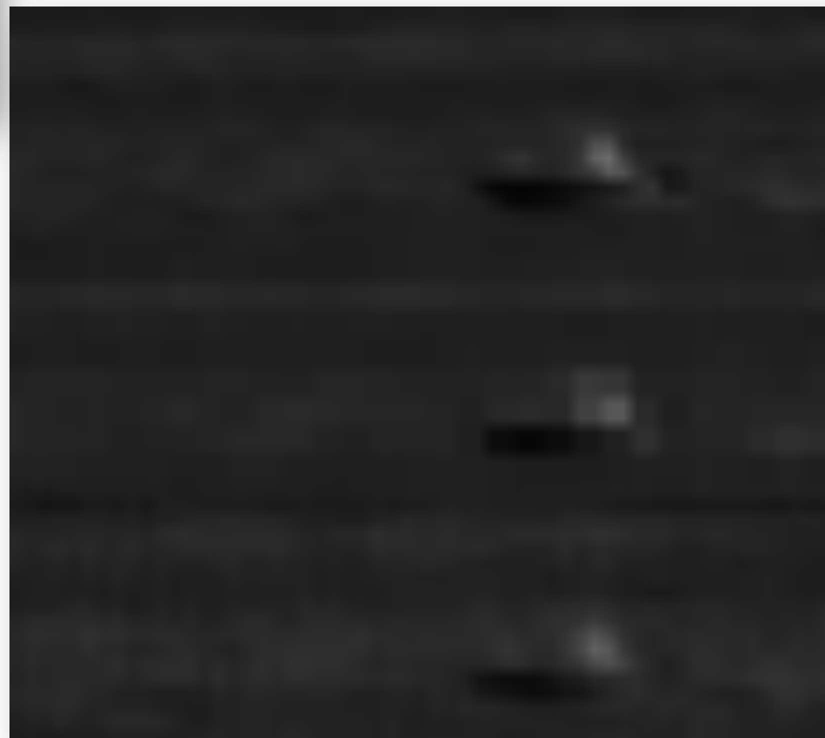
Original Scene

Original Scene Downsampled Reconstruction from coded observation

# RECONSTRUCTED VIDEO (FROM 2-FRAME METHOD)



Original Scene

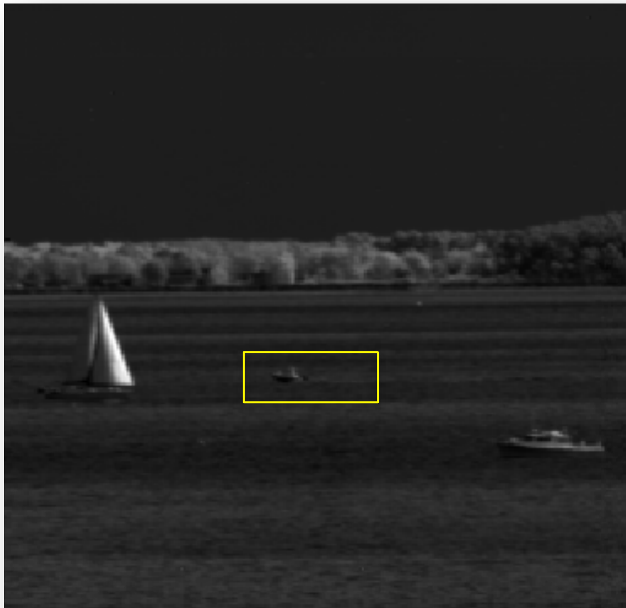


Original Scene

Downsampled

Reconstruction from coded observation

# Reconstructions of the 25th frame



Original Scene

Original Scene

Downsampled

Reconstruction from coded observation

# Concluding remarks

# Sparsity and Computing

- Sparsity plays a critical role in processing high-dimensional data
  - It increases our robustness to noise
  - It facilitates efficient storage and transmission of data
  - It allows us to fill in values of missing data
  - It helps us circumvent the curse of dimensionality and achieve accurate prediction performance
- Computational methods which exploit sparsity are
  - Fast
  - Sophisticated
  - Fun!

Shameless plugs

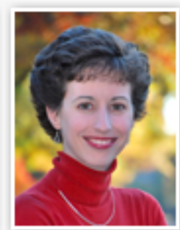
# Duke Workshop on Sensing and Analysis of High-Dimensional Data (SAHD)

The Duke University Workshop on Sensing and Analysis of High-Dimensional Data is planned for July 26-28, 2011. The meeting will be held at the Washington Duke Inn and Golf Club, adjacent to the Duke campus. The meeting is being organized and hosted by the following Duke faculty: David Brady, Robert Calderbank, Lawrence Carin, Ingrid Daubechies, David Dunson, Mauro Maggioni and Rebecca Willett.









The meeting is co-sponsored by AFOSR, AFRL, ARO, DARPA, NGA and ONR.

Rebecca Willett Assistant Professor, Duke University

Rebecca Willett Assistant Profes...

http://people.ee.duke.edu/~willett/

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# Rebecca Willett

Assistant Professor, Duke University
Electrical and Computer Engineering

Publications Research Software Teaching

## Research Interests

My research interests include network and imaging science with applications in medical imaging, wireless sensor networks, astronomy, and social networks. One central theme of my research is **data-starved inference for point processes** -- the development of statistically robust methods for analyzing discrete events, where the discrete events can range from photons hitting a detector in an imaging system to groups of people meeting in a social network. When the number of observed events is very small, accurately extracting knowledge from this data is a challenging task requiring the development of both new computational methods and novel theoretical analysis frameworks. This body of research has led to important insights into the performance of **compressed sensing** in optical systems, tools for tracking dynamic meeting patterns in **social networks**, predictions of future **IED locations** in Afghanistan, and novel sparse Poisson intensity reconstruction algorithms for **night vision** and **medical imaging**.

## Additional Activities

- Invited speaker at Workshop on Infusing Statistics and Engineering (WISE)
- 2011 Duke Workshop on Sensing and Analysis of High-Dimensional Data, July 26-28, co-organizer.
- Institute for Advanced Studies 2011 Program for Women and Mathematics Lecturer.
- Teaching course at 2011 Journées Statistiques du Sud.
- AFOSR Young Investigator Program recipient.
- NSF CAREER Award recipient.
- DARPA/IDA Computer Science Study Panel member; news article here.
- Duke Center for Theoretical and Mathematical Sciences
- Duke Network Analysis Center
- Duke Center For Metamaterials And Integrated Plasmonics Executive Advisor Board

**I hire students and postdocs from many backgrounds!**

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Have a nice day

Have a nice day